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ABSTRACT

The torsional deformation of aluminum and magnesium crystals is investigated, with particular reference to the dependence of proportional limit on crystal orientation. The proportional limit is found to be governed by the average value of the resolved shear stress on the most highly stressed slip systems, but the proportional limit does not strictly obey a critical-resolved-shear-stress law.

INTRODUCTION

The behavior of a cylindrical single crystal stressed in tension is well known: plastic flow begins when the resolved shear stress on the most highly stressed slip system reaches a critical value. There is much less understanding of the effect of torsion on such a crystal and on the way in which

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the critical-resolved-shear stress law should be applied to torsional deformation. In the present paper particular attention is paid to the dependence of plastic yielding on crystal orientation and lesser attention to the mechanism of flow after yielding has occurred. New work on aluminum and magnesium crystals is presented, and previous results obtained in this laboratory on magnesium[1] are reexamined.

STRESS DISTRIBUTION

The problems involved in torsional deformation of single crystals are due entirely to the complexity of the stress distribution. In tension, if loading conditions are perfect, the shear stress on any particular slip plane is completely uniform over the entire area of that plane. In torsion, on the other hand, the stress on any selected plane in a particular direction varies not only from center to edge of the specimen but also around its perimeter.

It is convenient to express the stress at any point in terms of τ_o, which is the shear stress acting at the surface of a cylindrical specimen on a plane normal to the axis and in a direction tangential to the cylinder. At point P of Fig. 1 this stress acts in the direction of axis 2 and is given by

$$\tau_o = \frac{2T}{\pi r^3} \qquad (1)$$

where $\underline{T}$ is the applied torque and $\underline{r}$ the specimen radius. This stress can be resolved on any chosen slip system, and details are given in the Appendix. Let $\underline{P}$ be the point on the specimen surface where the stress is to be evaluated; $\underline{P}$ is defined by its angular distance λ from an arbitrary reference line through $\underline{O}$ normal to the axis. (This line is usually taken as the direction of incidence of the X-ray beam used to determine the crystal orientation.) Then the shear stress τ_s acting at $\underline{P}$ in direction $\underline{D}$ on a plane whose normal is $\underline{N}$ is given by

$$\tau_s/\tau_o = \sin\theta_o \cos\theta_d \sin(\psi_o - \lambda) + \cos\theta_o \sin\theta_d \sin(\psi_d - \lambda) \qquad (2)$$

where θ_o and θ_d are the angles between $\underline{N}$ and $\underline{D}$, respectively, and the axis, and ψ_o and ψ_d are the angles between the projections of $\underline{N}$ and $\underline{D}$, respectively, on the transverse plane and the reference line through $\underline{O}$.

By means of Eq. (2) the stress τ_s acting in any given slip direction can be computed. In the previous work on magnesium[1] only gross slip tangential to the specimen surface was considered, and this slip was assumed to be governed by the tangential shear stress τ_t resolved parallel to the

slip plane and tangential to the surface, given by

$$\tau_t/\tau_o = \frac{\cos\theta_o\left[1-\sin^2(\psi_o-\lambda)\tan^2\theta_o\right]}{\sqrt{1+\sin^2(\psi_o-\lambda)\tan^2\theta_o}} \qquad (3)$$

The roles of τ_s and τ_t in determining the beginning of torsional yielding are discussed in detail later.

EXPERIMENTAL PROCEDURE

Most of the crystals were grown by the Bridgman method. They had a reduced center section, 1/4 inch in diameter and 2 inches long, and large end sections 1/2 inch in diameter. The impurity levels, according to the suppliers, were 0.004 pct for aluminum and 0.024 pct for magnesium.

To prepare the surface of the aluminum crystals for microscopic examination, they were mechanically polished through 3/0 metallographic emery paper and then chemically polished in a solution consisting of 80 parts ortho-phosphoric acid, 12 parts sulfuric acid, and 8 parts nitric acid at a temperature of 80-100°C.

The torsion machine and testing procedure have been previously described.[1]

RESULTS ON ALUMINUM

Plastic Yielding. Fig. 2 shows the orientations of the aluminum crystals, and Table 1 summarizes the test results on those crystals for which accurate torque-twist tests were made. In this table T_c is the critical torque required to initiate plastic flow, taken here as the proportional limit derived from the torque-twist curve, and τ_{oc} is the corresponding critical value of τ_o, obtained from Eq. (1).

The interpretation of these results can be best understood with reference to stress distribution plots like the one shown in Fig. 3. This shows the variation of the stress ratio τ_s/τ_o around the perimeter of the specimen for each of the twelve $\{111\}$ $\langle 110 \rangle$ slip systems of crystal Al-6. (The algebraic sign of the stress ratio is ignored in Fig. 3, and only one-half of the developed circumference is shown; the stress distribution on the other half is identical.) The numbers on each curve designate the slip system by a notation used by Gough *et al.*[2-6] The slip planes are numbered 0, 1, 2, and 3, and slip directions are defined by the intersection of two slip planes. The first integer in the symbol for a slip system identifies the plane, and the second integer the direction, of slip. Thus 03 designates slip on plane 0 in the direction of

intersection of planes 0 and 3.

Plots similar to Fig. 3 were made for the tangential stress ratio τ_t/τ_o. These contain only four curves per crystal, one for each of the slip planes.

The problem in applying the law of critical resolved shear stress to torsional deformation, i.e., the problem of establishing a relation between τ_{oc} and crystal orientation, is one of choosing a criterion for the beginning of observable plastic flow. Some possible criteria are listed below:

(1) Flow occurs when the maximum value of τ_s reaches the critical resolved shear stress $\underline{S}$. As the applied torque is increased from zero, $\underline{S}$ will first be reached, in general, at only one point on the half-circumference of the specimen. This criterion implies that the only significant feature of a plot like Fig. 3 is the maximum value of the stress ratio τ_s/τ_o, which will be denoted $\underline{F_{sm}}$. This quantity is a function only of crystal orientation.

(2) Flow occurs when the average value of τ_s, averaged around the specimen circumference, becomes equal to $\underline{S}$. The orientation factor for this criterion, denoted by $\underline{F_{sa}}$, is the average ordinate of the <u>envelope</u> of a set of τ_s/τ_o curves. This envelope is shown as a heavy line in Fig. 3.

(3) Flow occurs when the tangential stress τ_t at

a particular point on the slip plane perimeter becomes equal to S. This criterion was used by Hsu and Cullity[1] in their work on magnesium. They empirically determined the above mentioned point to be one which divides one quadrant of the slip plane perimeter into two portions: in one portion, comprising 0.68 of the perimeter, τ_t is smaller than S, and in the other 0.32 of the perimeter τ_t is larger than S. This criterion, involving an orientation factor F_{tp}, is inapplicable to any crystal having more than one slip plane, such as a face-centered cubic one.

(4) Flow occurs when the average value of τ_t, averaged around the specimen circumference, becomes equal to S. The orientation factor appropriate here, denoted by F_{ta}, is the average ordinate of the envelope of a set of τ_t/τ_o vs λ curves.

If the law of the critical resolved shear stress is valid in torsion, then the correct criterion for yielding will satisfy the following condition: the product of τ_{oc} and the orientation factor F will be constant and equal to S. Or

$$\tau_{oc} = \frac{S}{F} \qquad (4)$$

A plot of τ_{oc} vs. 1/F will therefore be a straight line through the origin with a slope of S.

If the line does not go through the origin, then no single critical stress, independent of crystal orientation, can be associated with yielding, and the relation becomes

$$\tau_{oc} = \frac{S}{F} + K \tag{5}$$

where the magnitude of the intercept K is a measure of the departure from a critical-resolved-shear-stress law.

The various kinds of orientation factors F are listed in Table 1 for the crystals tested. (Also listed are the F values appropriate to any face-centered-cubic crystal having its axis of torsion parallel to the stated crystallographic direction). Plots of τ_{oc} *vs* $1/F_{ta}$ or $1/F_{sm}$ showed no regularity whatever. However, the variation of τ_{oc} with $1/F_{sa}$ proved to be linear, as shown by Fig. 4. But the line does not go through the origin; it has an intercept K on the stress axis equal to 42.4 gm/mm^2. These results show that the average value of determines the beginning of yielding but that the law of critical resolved shear stress is not strictly followed.

Karnop and Sachs[7] came to similar conclusions in their study of the torsion of alloy crystals of aluminum containing 5 pct copper. The data in Table 1 of their paper, in which the "mean plastic

resistance" corresponds to $1/F_{sa}$, are plotted in Fig. 5. Although the points show considerable scatter, they fall approximately on a straight line with a _K_ intercept of -4.6 kg/mm^2. However, their values of τ_{oc} are derived, not from the torque at the proportional limit, but from the torque at an offset surface shear strain of 0.002. The latter value can be influenced by the rate of strain hardening, which depends on orientation in the same way as the yield torque (see next section) and would therefore tend to produce a negative intercept.

Plastic Flow. Gough _et al_.[2-6], studying crystals subjected to cyclically reversed torque, made a careful examination of the formation of slip lines on the surface. After 10^4-10^6 cycles at an appropriate maximum torque, they invariably found slip lines only in particular regions of the specimen circumference, i.e., only over particular ranges of the angle λ . Where the envelope of the τ_s/τ_o _vs_ λ curves reached a high value, slip lines appeared; where the envelope was low, no slip was observed. These findings support the critical-resolved-shear-stress law. They also show that localized slip in particular regions can occur and that this slip is governed by the value of τ_s in these regions, not by the average value of τ_s around the circumference.

In the present work, all crystals were examined with a microscope at 75X during torsion, and particular attention was paid to crystals Al-14 and 15. (No accurate torque-twist data could be obtained for these crystals, because they had to be removed from the torsion machine at intervals in order to permit the observed lines to be photographed.)

Slip lines were first observed at a torque somewhat higher than the proportional-limit torque $\underline{T_c}$. These lines were short, discontinuous, and did not appear at all points around the specimen circumference. At a still higher torque, traces of a second set of slip planes appeared: these lines were more closely spaced and were continuous around the crystal. The approximate torques at which this "first slip" and "second slip" were observed are listed in Table 1. Although most of the deformation seemed to occur on the second set of planes, as judged by slip line density, there was usually no change in the slope of the torque-twist curve when second slip was first observed, as a comparison of Table 1 and Fig. 6 will show. In a few crystals slip lines on a third set of planes were observed at high torques (Fig. 7).

A comparison of slip line inclinations, measured on photographs, crystal orientation, and computed stress-distribution curves led to the

following conclusions:

(1) Slip first appears in that slip system where τ_s is a maximum. Thus, in crystal Al-6 the $\{111\}$ plane designated 1 in Fig. 3 was the plane of first slip.

(2) Second slip appears in that slip system in which τ_s is second in magnitude, e.g., on plane O in crystal Al-6.

Thus the appearance of slip lines is governed by the maximum value of τ_s, whereas measurable plastic flow, defined as the point at which the torque-twist curve becomes non-linear, is determined by the average value of τ_s. Special efforts were made to discover whether or not measurable flow began inhomogeneously, i.e., at a point on the specimen where τ_s had its maximum value. Two cylindrical crystals, 1/2 inch in diameter, were grown by the strain-anneal method. Two sets of strain indicators were cemented to each specimen, one set at a circumferential position at which a maximum occurred in the τ_s/τ_o curve and the other set at the position of a minimum. Because of the large specimen diameter, the cemented joint between indicator and crystal extended over an angle λ of only about 5°. When these crystals were tested in torsion, each set of strain indicators led to the same proportional limit, thus supporting the view that the be-

ginning of measurable flow is governed by the average value of τ_s , at least as far as the sensitivity of these measurements is concerned. (The minimum detectable shear strain was 1.3×10^{-5} for the 1/4 inch diameter specimens and 2.9×10^{-5} for the 1/2 inch ones.)

Fig. 6 shows that the crystals may be divided into two groups according to their strain-hardening characteristics. Those whose axes lie near the 100-110 line in the unit stereographic triangle (Al-1, 4, 8, 6) have "hard" orientations while those nearer 111 are "soft" (Al-5, 2, 7). The crystals in the former group also exhibited more severe bending after considerable amounts of twisting.

RESULTS ON MAGNESIUM

Plastic Yielding. Fig. 8 shows the orientations of the magnesium crystals, and Table 2 summarizes the torsion test data obtained by Hsu and Cullity.[1] Orientation factors *F* of various kinds are also listed.

Figures 9 and 10 show plots of τ_{oc} *vs* 1/F. Straight lines are obtained, strangely enough, for all four orientation factors, and Table 3 lists the slopes and intercepts of these lines. (Hsu and Cullity gave 7.03 kg-mm as the value of T_c for crystal Mg-3, but inspection of their Fig. 2 shows that

the true value is probably closer to 4.0, which is the value used in Table 2 and Figures 9 and 10. Because of the uncertainty in T_c for this crystal the corresponding point was disregarded in drawing the lines shown in Figs. 9 and 10).

The small value of the intercept K found in the $1/F_{tp}$ plot shows that a critical-resolved-shear-stress law is quite closely followed, if it is assumed that yielding begins when the tangential stress $\tilde{\tau}_t$ reaches a critical value at a particular point on the slip plane perimeter. The critical stress S agrees with the value 38 gm/mm^2 obtained by Hsu and Cullity, as indeed it should. The intercepts on the other three plots ($1/F_{ta}$, $1/F_{sm}$, and $1/F_{sa}$) are all too large to permit description of the yield process in terms of a critical value of $\tilde{\tau}_{ta}$, $\tilde{\tau}_{sm}$, or $\tilde{\tau}_{sa}$.

Plastic Flow. Nine electrolytically polished crystals were examined with a microscope during torsion with the intention of discovering where slip first began. No evidence of localized slip at particular circumferential positions was found. As soon as slip was observed, the slip lines extended all around the elliptical perimeter of the slip plane. Gough and Cox[5] obtained somewhat similar results in their torsion fatigue tests of another hexagonal-close-packed metal, namely zinc. The chief difficul-

ty is that the slip direction is tangential to the surface, which makes slip detection difficult, at precisely those positions where the resolved stress τ_s has its maximum value. But Gough and Cox found minima in slip line density at positions where τ_s also went through minimum values, and their general conclusion was that slip in zinc, as in metals with other structures, was governed by the maximum value of τ_s at the position concerned.

An attempt to observe inhomogeneous yielding in magnesium was made in the same manner as for the aluminum crystals, i.e., two sets of strain indicators were attached to a 1/2 inch diameter magnesium crystal at positions differing widely in resolved stress τ_s . Both sets led to the same value of $\underline{T_c}$, even though the computed value of τ_s at one set of indicators was four times as large as that at the other set.

DISCUSSION

The great complexity of torsional deformation is attended by sources of experimental error more numerous than usual, and the possibilities of these errors should be kept in mind when evaluating the results. Loading conditions may deviate from ideality in at least two ways, and either may cause the computed stresses to deviate from the actual ones:

(1) A bending moment will be produced if the loading is not concentric.

(2) Axial stresses will be present if axial movement of the specimen ends is not permitted during torsion. (In the present work, no special attention was paid to the elimination of axial constraint. However, there was enough "slack" in the bearings of the torsion machine to permit some axial motion, and the results in the elastic range were probably unaffected. It is possible that this effect could have influenced the measured rates of strain hardening in the plastic range.)

Because of the very nature of torsional deformation it is difficult to detect initial yielding: the material which first becomes plastic comprises only a thin surface layer of the specimen. In contrast, yielding of a tensile specimen occurs across the entire section. An even more general difficulty arises from the non-uniform stress distribution in torsion: any flow of the surface layer is restrained by, and must accommodate itself to, the purely elastic deformation of the core. For this reason the computed surface stresses, and any yield criterion derived from them, probably have more applicability to a thin-walled tube than to a solid cylinder.

To first consider slip line formation, the work

of Gough et al. and the present results show, for a wide variety of crystal structures, that the first appearance of slip lines during torsion occurs where the resolved shear stress τ_s has its maximum value. Although it seems likely that slip begins at a particular point when τ_s reaches a critical value, none of the work mentioned has proved this point. Indeed, it would be most difficult to supply such proof, inasmuch as a definition of "initial slip" would involve the sensitivity of the means of observation and the inclination of the slip direction to the specimen surface. In the present work slip was observed only at torques above the proportional limit, but slip must, of course, have occurred at the proportional limit and probably, on a very fine scale, below it.

The dependence of proportional limit on crystal orientation has been examined in relation to several possible criteria for yielding. The criterion established by Hsu and Cullity for magnesium, involving a critical value of the tangential stress τ_t at a particular point on the specimen circumference, correlates the experimental results with considerable accuracy. However, this criterion must be rejected, not only because it has little or no physical significance, but also for the more compelling reason that it cannot be applied to crystals

which have more than one slip plane. Clearly, any acceptable criterion must be applicable to crystals of any structure. The only criterion found to be valid for both hexagonal-close-packed and face-centered-cubic crystals involves the average value of the resolved stress τ_s in the most highly stressed slip systems. However, the data do not strictly obey a critical-resolved-shear-stress law inasmuch as the plots of τ_{oc} vs $1/F_{sa}$ do not extrapolate to zero. What this criterion does permit is a prediction of the proportional limit of a crystal from a knowledge of the proportional limits of two other crystals of different orientations.

(In fairness to the data, it should be pointed out that the graphical method used here for presenting the measurements of proportional limit is a very severe test of the critical-resolved-shear-stress law. Fig. 11 summarizes all the data on an enlarged scale. It is obvious that slight changes in the experimental values can have an enormous effect on the intercept K because of the length of extrapolation; no 1/F values less than 1.0 are possible. On the other hand, a listing of the products $\tau_{oc} F_{sa} = S$ shows very little variation over the range of orientations tested and might be used to support a critical-resolved-shear-stress law. For example, these products have an average value, the "critical" value, for aluminum of 158

gm/mm^2, with a standard deviation of 2.1 gm/mm^2. For magnesium the average is 33.1 gm/mm^2 and the standard deviation 1.7 gm/mm^2. These "critical" values are also the slopes of the best straight lines drawn through the experimental points and the origin on plots of τ_{oc} vs $1/F_{sa}$).

The main differences between deformation in tension and in torsion may be summarized as follows. In tension, the beginning of slip and the departure from linearity of the load-elongation curve occur more or less simultaneously and at a critical stress. Of these two phenomena, slip is the more important because it is the cause of the observed proportional limit, which can be made evident by slip in only one slip system. In torsion, slip in the most highly stressed system probably begins at a critical stress. But slip in only one system cannot cause a proportional limit, because an overall twist of the specimen can be accomplished only by slip in several systems simultaneously. Therefore, a torque higher than that required to initiate slip in the first system must be applied before enough slip systems are operating to permit gross plastic twisting of the specimen and an observable proportional limit. This torque depends on crystal orientation through the average value of the shear stress resolved in the most highly stressed slip systems and is directly related to $1/F_{sa}$. How-

ever, the torque at the proportional limit is not determined by a critical stress. In view of the multiplicity of slip systems that must operate to produce a proportional limit, it is thus not surprising that proportional limits do not vary with orientation in strict accordance with a critical-resolved-shear-stress law.

In regard to the physical significance of the average stress τ_{sa} , which has been shown to be the connecting link between proportional limit and crystal orientation, it is possible only to speculate. In the first place, it must be remembered that a calculated stress distribution, such as that shown in Fig. 3, is valid only in the elastic region. As soon as slip begins, this distribution will be altered. Slip begins at those points on the specimen circumference where τ_s has its maximum values. This slip will tend to relieve the stress in these regions and increase it in others, inasmuch as the total moment of all the stresses must be constant and equal to the applied torque. This redistribution of stress is equivalent to a flattening out of the envelope of a set of τ_s vs λ curves, i.e., it leads, in the case of a perfectly uniform distribution, to the average value of the original envelope, namely τ_{sa} .

For magnesium, however, there is a serious quantitative difficulty with the notion of localized slip where τ_s is a maximum, followed by plastic yielding at some higher torque determined by the average value of τ_s . The critical resolved shear stress in tension of magnesium crystals prepared in the same way and from the same material as the torsion specimens was determined[1] to be 66 gm/mm^2. This is the stress required to activate a fairly large number of dislocation sources, and to cause continued expansion of dislocation loops, in a crystal subjected to a uniform shear stress. In a torsion specimen, the stress necessary to cause similar dislocation movement cannot be less and should be greater. (Because the shear stress decreases linearly from surface to axis in a torsion specimen, a dislocation line moving inwards finds itself in a decreasing stress field. It therefore tends to slow down and exert a back stress on other dislocations moving in behind it. As a result, the stress required to produce any considerable dislocation motion in a torsion specimen should be higher than in a tension specimen.) Yet it can be shown, from the data in Table 2, that the maximum value of τ_s at the proportional limit in torsion varies from about 40

to 60 gm/mm^2; these values are all less than the critical stress of 66 gm/mm^2 which produces slip in tension. And at torques somewhat below the proportional limit, when localized slip is assumed to begin, the maximum values of τ_s would be even smaller, approaching one half of the critical resolved shear stress in tension.

In aluminum, on the other hand, this difficulty does not arise. The maximum values of τ_s at the torsional proportional limit, computed from the data of Table 1, vary from 171 to 195 gm/mm^2. The critical resolved shear stress in tension, for crystals made by the same method (Bridgman) and of the same nominal composition as those tested in the present investigation, is reported to be 104 gm/mm^2 by Rosi and Mathewson[9] and 105 gm/mm^2 by Kramer and Maddin[10]. In this case, the stresses near which slip is assumed to begin in torsion are well above the corresponding stress for tension, as one would expect.

The torsional proportional limits of zinc crystals of various orientations have recently been measured by Brown and Rosenbaum[11]. They report these values in terms of the ratio τ_{oc}/τ_{ten}, where τ_{ten} is the critical resolved shear stress measured in tension. They plot this ratio against the angle ϕ between the specimen (torsion) axis

and the normal to the basal (slip) plane. That the experimental points fall fairly closely on a single curve must be considered fortuitous, because there is no physical justification for such a plot. The orientation dependence of the proportional limit must involve more than the single angle ϕ , especially in view of the fact that this angle has no significance for a crystal with more than one slip plane.

However, neither do their data conform to the criterion established in this paper for aluminum and magnesium. Stress distribution plots like those of Fig. 3 were made for each of the nine crystals tested by Brown and Rosenbaum, and values of F_{sa} were obtained from the envelope of each set of curves. Fig. 12 shows their data plotted in terms of τ_{oc}/τ_{ten} vs $1/F_{sa}$. The points do not fall on a single curve, much less on a straight line passing near or through the origin. The present writers are unable to account for the fact that these data on zinc do not conform to the criterion applicable to other metals, namely, that the proportional limit in torsion is governed by the average resolved stress in the most highly stressed slip systems.

CONCLUSIONS

The following conclusions apply to aluminum and magnesium single crystals:

(1) The initiation of slip at a particular point on the surface of a cylindrical crystal stressed in torsion is governed by the value of the resolved shear stress τ_s at that point.

(2) Plastic yielding in torsion, as measured by the proportional limit, is governed by the average value of τ_s in the most highly stressed slip systems, averaged around the specimen circumference. However, the proportional limit does not vary with orientation in strict accordance with a critical-resolved-shear-stress law involving this average value of τ_s .

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APPENDIX

Consider an elastically deforming cylindrical specimen subjected to a torque T (Fig. 1). The point P on the specimen surface is made the origin of a set of rectangular axes (1,2,3), where 1 is normal to the surface, 2 is tangential to the transverse section, and 3 is parallel to the axis. Then at P only two non-zero members of the stress tensor remain:

$$\tau_{23} = \tau_{32} = \tau_0 = \frac{2T}{\pi r^3} \qquad (6)$$

where r is the specimen radius. Wright[8] has shown that this analysis is valid for both elastically isotropic and anisotropic materials, since the same compatibility equations must be obeyed in each case.

To determine the shear stress resolved in a particular slip system, transform the axes (1,2,3) to a new set (1',2',3') where 2' is parallel to the slip direction D and 3' is parallel to the slip plane normal N. These new axes are shown in Fig. 13, where the reference line RP is a line parallel to the radius through O in Fig. 1.

In general, for a change of axes the transformation law for tensors may be written

$$K'_{jl} = l_{ij}\, l_{kl}\, K_{ik}$$

where K'_{jl} is the tensor referred to the primed axes, K_{ik} the tensor referred to the unprimed axes, and l_{ij} the cosine of the angle between the ith unprimed axis and the jth primed axis (l_{kl} is defined similarly). The repetition of the indices i and k implies summation with respect to these indices. Since, in the present case, the only non-zero elements of the stress tensor are τ_{23} and τ_{32}, the shear stress resolved in the slip system defined by the primed axes is

$$\tau_s = \tau_{2'3'} = l_{22'} l_{33'} \tau_{23} + l_{32'} l_{23'} \tau_{32} \tag{7}$$

The direction cosines, determined by vector analysis involving unit vectors along each of the primed and unprimed axes, are

$$l_{33'} = \cos\theta_o$$
$$l_{32'} = \cos\theta_\ell$$
$$l_{22'} = -\sin\theta_\ell \sin(\psi_o - \lambda)$$
$$l_{23'} = -\sin\theta_o \sin(\psi_o - \lambda)$$

Combination of these relations with Equations (6) and (7) gives

$$\tau_s/\tau_o = \sin\theta_o \cos\theta_\ell \sin(\psi_o - \lambda) + \cos\theta_o \sin\theta_\ell \sin(\psi_\ell - \lambda) \tag{8}$$

The tangential stress τ_t equals τ_s when

$$\psi_\ell = 90^\circ + \lambda \tag{9}$$

The condition that D lie in the slip plane whose normal is N is

$$\cot\theta_d = -\tan\theta_0 \cos(\psi_d - \psi_0) \qquad (10)$$

Combination of Equations (8), (9), and (10) gives

$$\frac{\tau_E}{\tau_0} = \frac{\cos\theta_0\left[1 - \sin^2(\psi_0 - \lambda)\tan^2\theta_0\right]}{\sqrt{1 + \sin^2(\psi_0 - \lambda)\tan^2\theta_0}} \qquad (11)$$

This equation is equivalent to Equation (2) of Hsu and Cullity[1] when allowance is made for differences in notation. The notation of the present paper is that of Gough[4] and is related to that of Hsu and Cullity by the following equations:

$$\theta_0 = \phi$$

$$(\psi_0 - \lambda) = -(\theta + 90^\circ)$$

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TABLE 1. TORSIONAL PROPORTIONAL LIMITS AND ORIENTATION FACTORS OF ALUMINUM CRYSTALS

Specimen	r (mm)	T_c (kg-mm)	τ_{oc} (kg/mm^2)	Orientation Factor F_{ta}	F_{sm}	F_{sa}	Torque (kg-mm) at observed slip: First	Second
Al-1	2.87	6.85	184	0.75	0.98	0.87	7.3	18.5
Al-2	2.87	6.70	180	0.85	0.95	0.87	8.6	16.3
Al-4	2.87	6.60	177	0.70	1.00	0.91	12.4	20.1
Al-5	2.81	6.50	186	0.84	1.00	0.84	12.4	18.8
Al-6	2.65	5.10	174	0.60	0.99	0.91	12.4	22.6
Al-7	2.60	4.80	174	0.96	0.98	0.92	9.9	15.0
Al-8	2.72	6.40	203	0.48	0.96	0.75	8.6	22.6
100				0.33	0.58	0.52		
110				0.88	1.00	0.89		
111				1.00	1.00	0.96		

TABLE 2. TORSIONAL PROPORTIONAL LIMITS AND ORIENTATION FACTORS OF MAGNESIUM CRYSTALS

Specimen	r (mm)	T_c (kg-mm)	(gm/mm^2)	Orientation Factor F_{tp}	F_{ta}	F_{sm}	F_{sa}
Mg-9	3.09	2.21	47.6	0.76	0.67	0.84	0.66
Mg-1	3.03	2.23	51.1	0.72	0.63	0.81	0.62
Mg-8	3.11	2.90	61.3	0.60	0.50	0.78	0.55
Mg-6	3.10	4.06	86.9	0.43	0.32	0.69	0.41
Mg-3	3.18	4.0	79.1	0.23	0.28	0.61	0.42

TABLE 3. SLOPES AND INTERCEPTS OF CURVES IN FIGS. 4, 5, 9, and 10

Material	Stress Factor	Slope S (gm/mm^2)	Intercept K (gm/mm^2)
Al-5 Cu	F_{sa}	13.61*	-4.6*
Al	F_{sa}	121.2	42.4
Mg	F_{tp}	38.6	-3.0
Mg	F_{ta}	23.7	13.3
Mg	F_{sm}	150.0	-131.0
Mg	F_{sa}	42.7	-17.1

*The values for the Al-Cu alloy are in kg/mm^2.

TORSIONAL DEFORMATION OF IRON SINGLE CRYSTALS

C. W. Allen and B. D. Cullity

ABSTRACT

The proportional limit of iron crystals in torsion is governed by the resolved shear stress in the most highly stressed slip systems, averaged around the specimen circumference, and does not obey a critical-resolved shear-stress law. Crystals of most orientations exhibit a stage of easy plastic deformation, akin to easy glide in tensile or shear specimens. Transient deformation, similar to that which occurs in single crystals of other materials, is also observed.

INTRODUCTION

The torsional deformation of single crystals of magnesium (HCP) and aluminum (FCC) has been described recently by Choi *et al.*,[1] especially with respect to the criterion for the orientation dependence of the onset of plastic flow in these materials. The purpose of this paper is to present results of torsion tests of iron single crystals and thus to extend this yield criterion to a BCC metal. In addition to considering the variation of proportional limit with

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crystal orientation, this paper also briefly treats work hardening, transient deformation, and the mechanism of plastic flow in iron. The effects of the method of surface polishing and the chemical purity of the iron have been investigated.

STRESS DISTRIBUTION

It is convenient to express the stress at any point of a cylindrical crystal stressed in torsion in terms of τ_o , which is the shear stress acting at the surface on a plane normal to the axis of the cylinder and in a direction tangential to the cylinder. This stress is given by

$$\tau_o = \frac{2T}{\pi r^3} \tag{1}$$

where T is the applied torque and r the specimen radius. The shear stress τ_s resolved in any chosen slip system is given in terms of τ_o by[1]

$$\tau_s/\tau_o = \sin\theta_o \cos\theta_d \sin(\psi_d - \lambda) + \cos\theta_o \sin\theta_d \sin(\psi_d - \lambda) \tag{2}$$

where θ_o and θ_d are the angles between the specimen axis and the slip plane normal and slip direction, respectively; λ is the angular circumferential position on the specimen at which τ_s is being determined, measured from an arbitrary reference plane which includes the axis; ψ_o and ψ_d are the angular coordinates of the projections of the slip plane normal and

slip direction on a transverse section with respect to this same reference plane.

Slip in iron occurs in a $\langle 111 \rangle$ direction on the $\{110\}$, $\{112\}$, and $\{123\}$ planes, which together comprise 48 slip systems. A complete evaluation of the stress distribution in an iron crystal stressed in torsion would therefore require a calculation of τ_s/τ_o as a function of λ for 48 different slip systems. Fortunately Gough[2], who studied the behavior of iron crystals in alternating torsion, was able to simplify this problem considerably. He showed that it was sufficient to consider a kind of average slip plane for each slip direction, namely the mathematical plane of maximum resolved shear stress containing the slip direction considered. This simplifying approximation is possible because, for each slip direction, the <u>active</u> slip plane or planes lie very near this mathematical plane of maximum shear stress.

Vogel and Brick[7] have critically reviewed the early work of Taylor and Elam[13], Taylor[14] and Fahrenhorst and Schmid[8] from which the identification of the above crystallographic planes as slip planes in the BCC lattice largely stems. While their criticism is clearly justified, their own results do little to clarify the issue. The role of cross slip (screw dislocations changing glide

planes) is evidently so important in this case, as Read[3] has suggested, that methods for deducing slip systems from observations of gross slip traces are inadequate, such traces commonly arising from complex dislocation motion. Thus the treatment given here involving the plane of maximum resolved shear stress seems a logical simplification especially in view of Gough's[2] study of alpha-iron. There is, however, an assumption built into the subsequent treatment the comparative validity of which is difficult to assess, namely, that slip in all slip systems in iron may be characterized by a common critical resolved shear stress.

The shear stress τ_m resolved in a slip direction defined by θ_d and ψ_d, and on the plane of maximum shear stress containing this direction, is found by first maximizing τ_s/τ_o with respect to either θ_o or ψ_o. The slip plane coordinates are then eliminated by using the relation between θ_o, ψ_o and θ_d, ψ_d, namely,

$$\cot\theta_d = -\tan\theta_o\cos(\psi_d - \psi_o).$$

The result, substituted back into Eq. (2), gives

$$\tau_m/\tau_o = \left[\cos^2\theta_d\cos^2(\psi_d-\lambda) + \cos^2 2\theta_d\sin^2(\psi_d-\lambda)\right]^{1/2} \tag{3}$$

Use of this relationship instead of Eq. (2) reduces the number of calculations involved in the shear stress analysis of one crystal from 48 to 4.

It is important to note that the plane to which τ_m refers (the mathematical plane of maximum shear stress) is not a plane of low indices or of invariable indices. Actually, the indices of this plane vary from point to point around the specimen. Thus, τ_m for a particular slip direction, say $[111]$, at a point on the specimen circumference defined by a particular value of λ , refers to a plane containing $[111]$ on which τ_s is a maximum. This plane will have nearly the same orientation as some one plane of the set of 12 planes $\{110\}$, $\{112\}$, $\{123\}$, containing $[111]$; accordingly, τ_m will nearly equal the stress on the most highly stressed plane of the 12 planes containing $[111]$. This will be true for any value of λ , but the indices of the plane to which τ_m refers will change with λ . Fig. 1 shows how τ_m/τ_o varies with λ for each of the four $\langle 111 \rangle$ slip directions in crystals of three particular orientations.

EXPERIMENTAL PROCEDURE

Single crystals were produced by the strain-anneal method. Armco iron rod, 1/4 inch in diameter, containing 0.022 pct carbon and 0.183 pct other impurities, was swaged to various diameters and subsequently fully annealed in vacuum for 1 hour at 950°C. Each specimen was pulled to a permanent strain of

0.02, and groups of twenty specimens were subjected to a programmed anneal consisting of a continuous temperature increase from 540 to 850°C at a rate of 80°C per day. The specimens were held at 850°C for 4 days, and then furnace cooled. An atmosphere of dry, deoxidized helium was employed. The specimens were etched in nitric acid to remove the fine whisker-like coating of magnetite and ferrite and to determine quickly which of the samples were single crystals. The orientations of these were then determined by the back-reflection Laue method. In Fig. 2 the axes of the crystals tested are plotted in a stereographic triangle.

With respect to surface preparation, the specimens fall into three categories:

(1) Chemically polished only. (The solution is that recommended by Beaujard[4] and consists of 3 parts HNO_3, 7 parts HF, and 30 parts H_2O. Specimens were polished for 2 minutes at 60°C with no agitation.)

(2) Mechanically polished (through 3/0 metallographic emery paper) and then chemically polished.

(3) Mechanically polished and then electrolytically polished. (Electrolyte: 56 pct solution, by volume, of phosphoric acid in water. Voltage: 0.15-0.20. Current density: 0.01 amp/cm^2.)

One group of specimens was given a purifying anneal in hydrogen after removal from the crystal-growing furnace and before polishing. These specimens

were annealed at 850°C in a static, undried, hydrogen atmosphere for one week, then in vacuum for one hour, and furnace cooled.

A special back-reflection Laue X-ray method, to be described in detail elsewhere, was devised for the purpose of assessing relative crystal perfection. It was used to study crystal surfaces both before and during the torsion tests.

The torsion tests were performed in the machine previously described[5], but modified as follows. Three additional pulleys, over which the load is applied to the circular torsion arm, were added to permit application of a pure couple and reversal of the direction of the twisting moment. (The results of such forward-reverse torsion tests on single crystals of iron and other metals will be reported elsewhere.) The upper bearing of the machine was re-designed to remove any constraint on the free elongation of the specimen during torsion that might previously have existed. The torsional strain was measured over a gage length of 1 1/4 in. It should be noted that the tests were essentially static in character; i.e., they were performed by increasing the torque in small increments, after each of which the resultant angle of twist was determined when any transient plastic deformation had subsided.

RESULTS

<u>Crystal Perfection</u>. The special Laue method was applied to quartz, chosen as a standard "perfect" crystal, and to various iron crystals. This method is capable of revealing angular disorientations between subgrains of the order of a few minutes of arc over a considerable area of specimen surface. Within this limit of sensitivity, the etched, as-grown iron crystals appeared to approach the perfection of the quartz standard. However, mechanical polishing, even though followed by electrolytic polishing, was found to produce disoriented material in the surface extending over several minutes of arc. This alteration of the surface was later found to have an effect on the torsional behavior of the crystal.

<u>Plastic Yielding</u>. Choi *et al.*[1] found that the torsional proportional limit of aluminum and magnesium crystals was governed by the average value of τ_s over the circumference of the specimen. For iron crystals the average value of τ_m can be expected to have similar significance. Stress distribution curves similar to those of Fig. 1 were therefore calculated for each crystal. The average value of the envelope of each set of τ_m/τ_o *vs* λ curves was then computed; this quantity, the orientation factor, is

designated F_{ma} and is listed for each crystal in Table 1. Also tabulated are the values of $\underline{T}_c$ and τ_{oc} , which are the values of $\underline{T}$ and τ_o , respectively, at the observed proportional limit. Fig. 3 shows typical shear stress-shear strain curves. These were obtained from the experimental torque-twist curves by computing the shear stress τ_o from Eq. (1) and the shear strain γ from

$$\gamma = \frac{\alpha r}{\ell}$$

where α is the angle of twist in the gage length ℓ . The shear stresses τ_o so computed are entirely nominal above the proportional limit, because Eq. (1) is valid only in the elastic range, but they permit presentation of data on crystals of different radii on the same basis.

If yielding, as defined by the proportional limit, occurs when the average value of τ_m reaches a critical value $\underline{S}$, then the product $\tau_{oc}\,\underline{F_{ma}}$ will be constant and independent of crystal orientation, or

$$\tau_{oc} = \frac{S}{F_{ma}}$$

A plot of τ_{oc} $\underline{vs}$ $1/F_{ma}$ will therefore be a straight line through the origin with a slope of $\underline{S}$. If the line does not go through the origin, then

$$\tau_{oc} = \frac{S}{F_{ma}} + K$$

and the magnitude of the intercept K is a measure of the departure from a critical-resolved-shear-stress law. Fig. 4 shows plots of τ_{∞} vs $1/F_{ma}$ for three sets of differently treated crystals. The points fall fairly accurately on straight lines, but these lines do not pass through the origin. Their slopes and intercepts are listed in Table 2.

Karnop and Sachs[6] suggested that there might be a size effect for the torsion of single crystals because of a pronounced interaction of the various active slip systems assumed to occur in smaller specimens. This view implies that T_c, the torque at the proportional limit, would not vary linearly with the cube of the specimen radius, as Eq. (1) states. This view was tested for a set of specimens which had similar orientations, namely, axes near $\langle 110 \rangle$, and therefore almost identical values of the orientation factor F_{ma}, but whose radii varied from 1.30 to 2.53 mm. The data for these specimens are plotted as curves 2 and 4 of Fig. 5, which shows that T_c is proportional to r^3. Furthermore, these curves extrapolate to a common value of T_c of 1.0 to 1.5 kg-mm at $r = 0$, which is very close to the experimentally determined value of 0.7 kg-mm for the static friction of the torsion machine. (Points for two specimens deviate considerably from curve 2. These deviations will be discussed later.)

Attempts were made to find evidence of localized yielding at points on the specimen surface where τ_m had a maximum value. No such evidence was found, either by the special X-ray method referred to above or by the double strain indicator technique used in the study of aluminum and magnesium.[1]

Work Hardening. Inspection of Fig. 3 shows wide variations in the rate of work hardening after plastic flow begins. Many specimens, especially those with orientations near $\langle 110 \rangle$ and $\langle 111 \rangle$, exhibit a range of extensive deformation with practically no increase in applied torque. This effect resembles easy glide in tension but will be called "easy deformation," because it involves the simultaneous action of several slip systems, at different points within the specimen, whereas easy glide refers to extensive slip in one system only. The critical torques T_c and corresponding stresses τ_{oc} for easy deformation are listed in Table 1. It was found that the values of T_c for easy deformation of crystals of similar orientation are also proportional to r^3, as curves 1 and 3 of Fig. 5 show.

Transient Deformation. As in previous work[1] on magnesium and aluminum, transient effects were found in the torsion of iron crystals, i.e., after a given increment of torque, the angle of twist reaches its final value not instantaneously but only after the

elapse of some time. Typical behavior of this kind is illustrated in Fig. 6. If the torque increment is not to large, the rate of twisting is initially very low, then it increases, and finally decreases practically to zero.

DISCUSSION

The fact that straight lines are obtained in Fig. 4 demonstrates that the torsional proportional limit of similarly prepared iron crystals is governed by the average value of τ_m around the circumference of the specimen. But the relatively large values of the intercepts of these lines (Table 2) show that the proportional limit is not determined by a *critical* value of the τ_m average. Thus it is necessary to know the proportional limits of not one but two crystals of different orientations before the proportional limit of a similarly prepared third crystal can be predicted. In this respect the behavior of iron resembles that of aluminum and magnesium.[1]

The results on these three metals, which run the gamut of the common crystal structures, therefore show that gross yielding in torsion is governed by the resolved shear stress in the most highly stressed slip systems, averaged around the specimen circumference.

Unlike magnesium, however, for which the resolved shear stress at the proportional limit is about half that required to initiate yielding in a tensile test, iron exhibits approximately the same resolved shear stress at yielding in both torsion and tension. The tensile yield stress for an as-grown iron crystal, the orientation of which is near $\langle 110 \rangle$, was found to be 4.37 kg/mm^2; the corresponding resolved shear stress was 2.19 kg/mm^2, a value low compared with the results of Vogel and Brick[7] and of Fahrenhorst and Schmit[8], who report 4.31 and 4.45 kg/mm^2, respectively, but in substantial agreement with the results of torsion tests of comparably prepared crystals for which the maximum values of $\tilde{\tau}_m$ at the torsional proportional limits ranged from 1.85 to 3.84 kg/mm^2 (it will be remembered that $\tilde{\tau}_m$, at any given torque, varies in magnitude around the specimen circumference). No yield point was observed for the tensile test specimen at a strain rate of 0.005 per minute.

Relative chemical purity, attained by annealing in hydrogen, has little effect on proportional limits, provided there has been no mechanical polishing. This is shown by the fact that curves 1 and 2 of Fig. 4 have about the same slope and intercept values. The data of Fig. 5 also support this conclusion. Here the points representing the proportional limits of specimens I-12 and I-15 (not annealed in hydrogen,

chemically polished only) lie well below curve 2 and near curve 4, which represents hydrogen-annealed specimens. Mechanical polishing, even though followed by chemical polishing, apparently hardens the surface layer to such an extent that the proportional limit is raised nearly to the easy deformation point; thus the points in Fig. 5 representing specimens E-4, E-7, and I-11 (all mechanically polished) all fall on curve 2, which lies just below easy deformation curve 1.

The insensitivity of the proportional limit of chemically polished specimens to hydrogen annealing is completely consistent with the calculations which Cottrell and Bilby[9] have made on dislocation-interstitial interactions and the yield point in iron. According to them, a very low impurity content (10^{-4} weight pct C for a dislocation density of $10^8/cm^2$) is enough to form an atmosphere of solute atoms along each dislocation line and restrain its glide. The impurity content of the specimens tested in this investigation would certainly be above this level, whether hydrogen annealed or not.

However, relative purity does have a pronounced effect on the torque required for easy deformation, as shown in Fig. 5 by the wide separation between curve 1 (no hydrogen anneal) and curve 3 (hydrogen anneal). Where the torque at easy deformation is

high, e.g., specimen I-15 in Fig. 3, the easy deformation is usually preceded by very severe, almost linear, strain hardening. Specimens exhibiting this behavior are relatively impure (not annealed in hydrogen), and their stress-strain curves resemble curves obtained by Wiseman, Parker, and Hazlett[10] during shear tests of copper single crystals containing 1 to 15 atom pct of solute elements. According to these investigators, the steep portion of the curve corresponds to the motion of dislocations from surface sources to sub-grain boundaries, where they become stuck; the essentially flat portion is reached when the stress is high enough to free dislocations from sub-boundaries, after which they move easily through the lattice and other sub-boundaries. A similar explanation would seem to be applicable here, despite the large difference in "impurity" content. In addition, because of the comparatively high percentage of carbon in the impure crystals, probably a fine dispersion of cementite is present within the ferrite crystals, a situation which is probably not present in the crystals subjected to the long time hydrogen annealing treatment.

Fig. 3 shows the peculiar behavior of hydrogen-annealed specimens (I-2, I-3, and I-8) above the proportional limit. There is a stage of easy deformation, followed by a region of almost elastic be-

havior, and then easy deformation again. The reason for this elastic region is not clear, but it may be due to a relatively impure core in the specimen. Dislocations held up by this core would build up a back stress sufficient to prevent the operation of sources at or near the surface, until the applied stress becomes high enough to allow the dislocations to break into, and move through, the impure core.

The phenomenon of easy deformation in torsion depends on crystal orientation in somewhat the same way as easy glide does in tensile and shear tests. Easy glide occurs only when the specimen is oriented for the operation of a single slip system. In torsion the situation is more complicated, because several slip systems must always operate together to produce any overall twist. The important criterion in torsion seems to be whether or not a single slip system is operating at a particular point. (Here "slip system" really means "slip direction" because of the quasi-indeterminacy of the slip plane in iron.) Thus, easy deformation was found to be much less extensive in crystals with axes near $\langle 100 \rangle$ than in crystals of other orientations. Inspection of Fig. 1 shows that at least two slip systems can operate simultaneously at any one point on the perimeter of a specimen with a $\langle 100 \rangle$ orientation. This condition is conducive to rapid work hardening. But

in crystals deviating from this orientation one slip system is definitely favored over the other three at practically every point of the specimen perimeter, although it must be remembered that different slip systems operate .n different sectors of the perimeter. Such crystals will exhibit easy deformation. Thus, easy deformation can be said to be favored whenever easy glide can occur in various sectors of the specimen.

That the torque required for easy deformation is quite accurately proportional to $\underline{r}^3$ is indeed surprising. (Fig. 5). Some plastic flow has occurred before easy deformation begins, and yet Eq. (1) is valid only in the elastic region.

As to the transient effects here observed in torsion (Fig. 6), similar curves for tension have been found by Gensamer and Mehl[11] for iron crystals and by many others for various metals. Such curves have also been reported for the creep of sapphire by Wachtman and Maxwell.[12] The various changes in the observed flow rate may be explained as follows. The first stage of very slow flow is associated only with essentially reversible dislocation line undulations. The presence of this initial stage indicates that this transient deformation is not due significantly to an anelastic relaxation of solute impurities. This "incubation period" is followed by a stage of rapid

flow, in which an ever increasing number of dislocations become free to move and/or originate from dislocation sources; both events are aided by the stress field of already moving dislocations. In the third stage work hardening sets in, due to exhaustion and interactions, and the flow rate drops almost to zero. The flow rate is then presumably controlled by thermally activated cross slip of screw dislocations and possibly by dislocation climb and, as in the first stage, by thermal undulation of pinned dislocation lines, released from time to time and driven by the local stress field.

CONCLUSIONS

(1) The torsional proportional limit of iron crystals (BCC), like that of aluminum (FCC) and magnesium (HCP), is governed by the resolved shear stress in the most highly stressed slip systems, averaged around the specimen circumference. However, the proportional limit is not governed, in any of these metals, by a critical-resolved-shear-stress law.

(2) Most iron crystals exhibit a stage of easy deformation. The tendency toward easy deformation depends on orientation and the stress at which it begins, on purity. The proportional limit of similarly oriented and prepared crystals is insensitive to purity.

(3) Iron crystals in torsion exhibit transient deformation effects similar to those shown by other metallic

and non-metallic crystals in tension.

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TABLE 1 - SUMMARY OF TORSION TEST RESULTS

Specimen	Surface	r (mm)	Proportional Limit T_c (kg-mm)	Proportional Limit τ_{oc} (kg/mm^2)	Easy Deformation T_c (kg-mm)	Easy Deformation τ_o (kg/mm^2)	F_{ma}
E-3	M, E	1.40	20.1	4.65			0.680
E-4	M, E	1.30	13.5	3.96			0.882
E-7	M, E	1.42	20.3	4.52			0.860
I-2-H	M, C	2.46	52.6	2.25			0.891
I-4-H	M, C	1.91	32.7	3.00	32.8	3.01	0.812
I-6-H	M, C	2.05	28.5	2.12	32.8	2.44	0.897
I-11	M, C	2.11	57.3	3.88	62.0	4.25	0.870
F-2	C	1.34	9.2	2.44	11.7	3.10	0.823
F-10	C	1.42	10.7	2.37	11.5	2.54	0.852
I-3-H	C	2.36	38.9	1.88	49.0	2.37	0.921
I-5-H	C	2.07	36.4	2.61	45.0	3.22	0.694
I-8-H	C	2.24	50.1	2.85	60.3	3.43	0.650
I-9	C	2.55	52.1	2.00	86.9	3.34	0.985
I-10	C	1.94	33.6	2.84			0.861
I-12	C	2.53	59.1	2.41	103.4	4.07	0.847
I-13	C	1.88	35.3	3.38			0.645
I-15	C	1.89	23.1	2.20	44.5	3.99	0.890
I-16	C	2.35	47.6	1.87			0.987

Notes:

(a) In column 1, specimens annealed in hydrogen have the letter H after the specimen designation.

(b) In column 2, the letters M, C, and E designate mechanical, chemical, and electrolytic polishing, respectively.

(c) All values of T_c and τ_o have been corrected for 0.7 kg-mm of machine friction.

(d) The values of F_{ma} when the torsion axis is parallel to an important crystallographic direction are 0.538 for $\langle 100 \rangle$, 0.888 for $\langle 110 \rangle$, and 1.000 for $\langle 111 \rangle$.

Table 2. Slopes and Intercepts of Lines in Fig. 4

Line	Slope S (kg/mm^2)	Intercept K (kg/mm^2)
1	2.18	- 0.50
2	2.36	- 0.70
3	7.47	- 6.24

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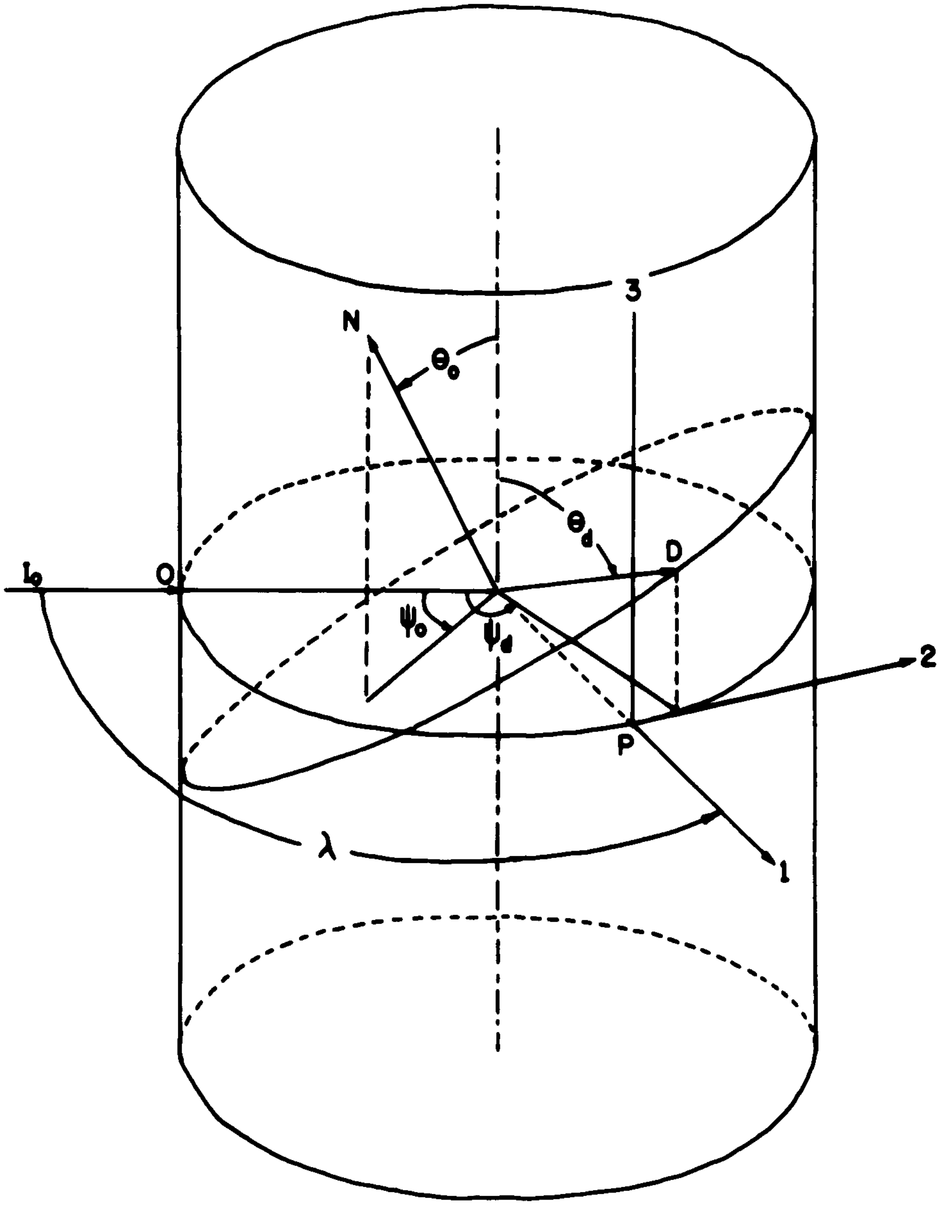

Fig. 1. Cylindrical torsion specimen.

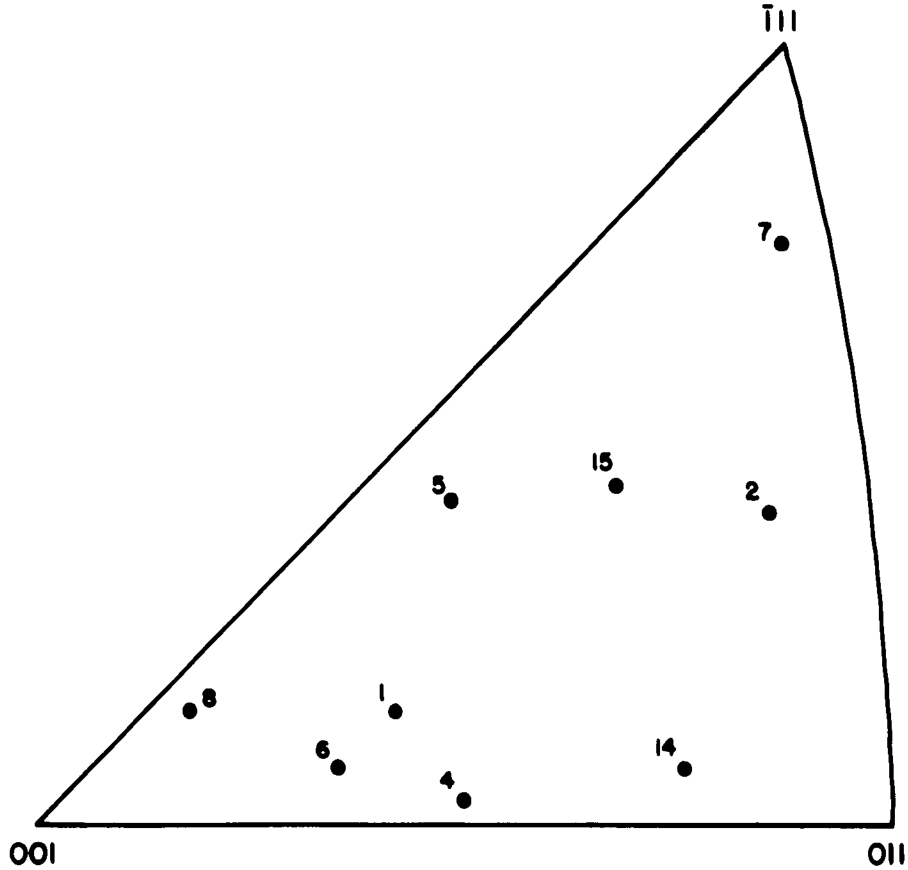

Fig. 2. Orientations of aluminum crystals.

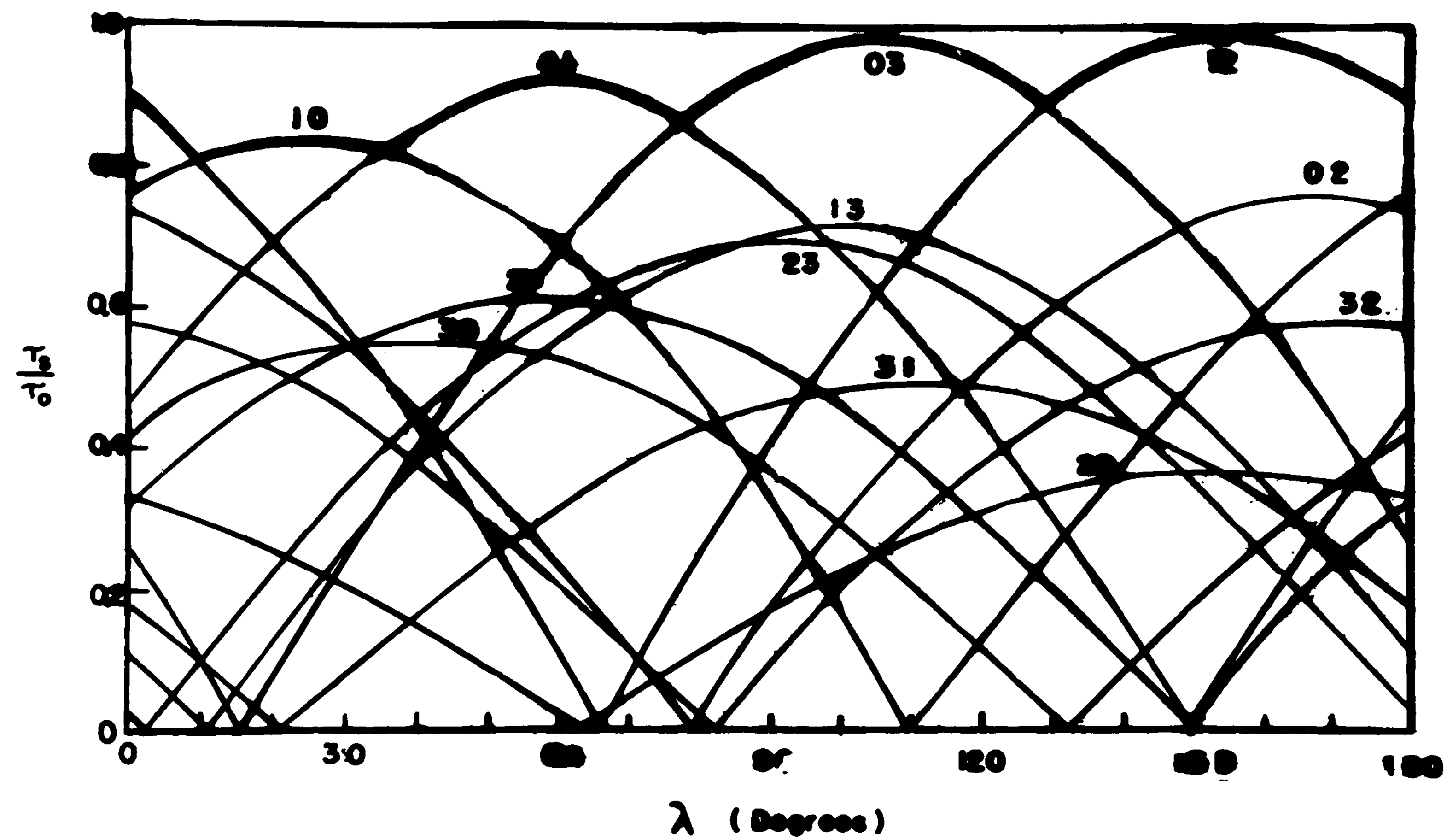

Fig. 3. Calculated stress distribution for crystal Al-6.

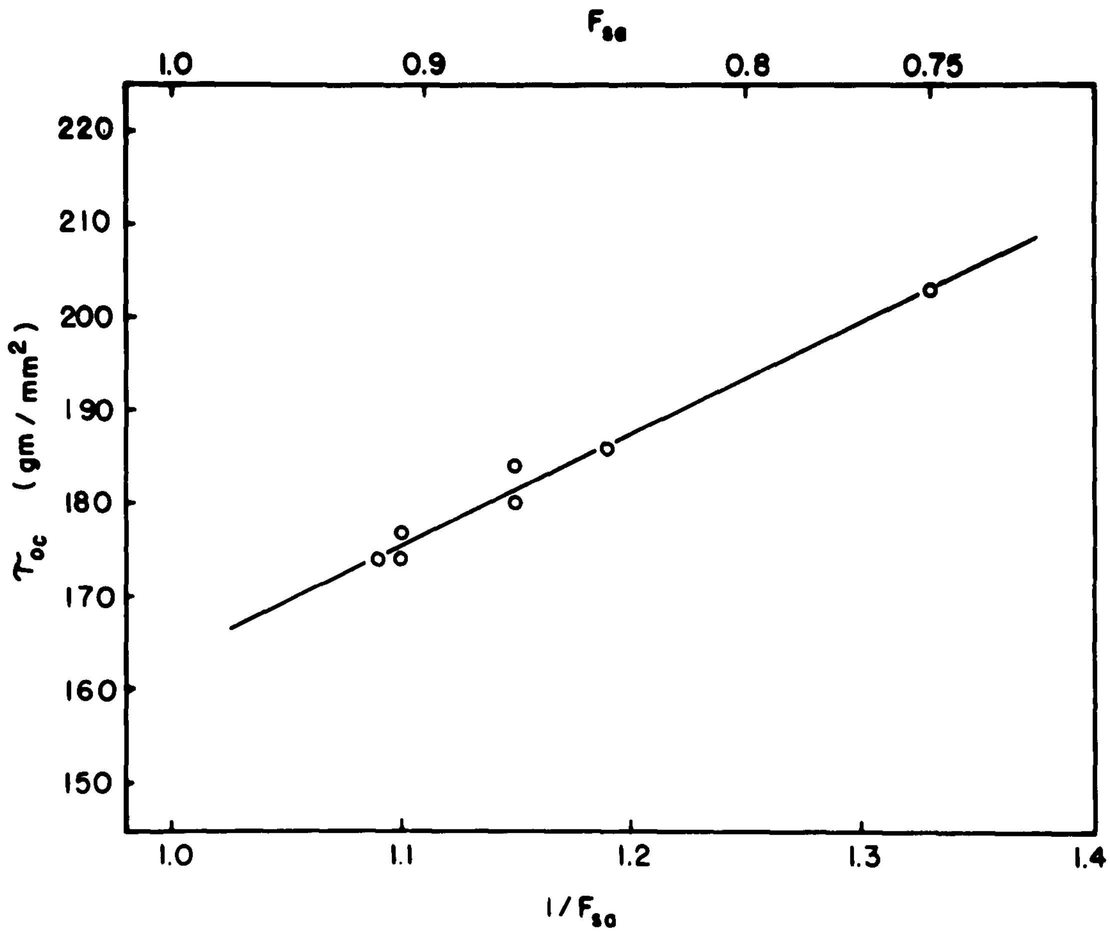

Fig. 4. Dependence of plastic yielding on orientation factor F_{sa} for aluminum crystals.

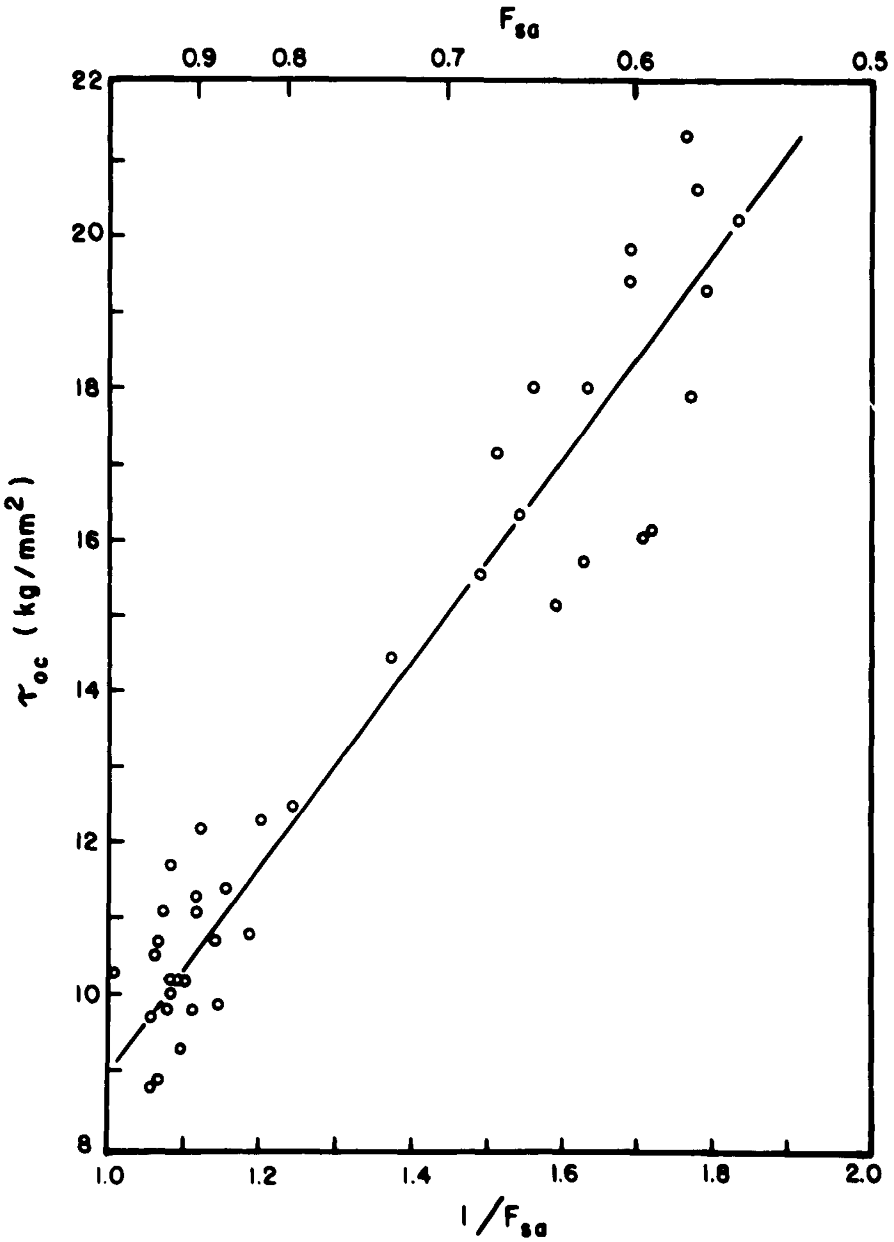

Fig. 5. Dependence of plastic yielding on orientation factor F_{sa} for the aluminum – copper crystals tested by Karnop and Sachs.[7]

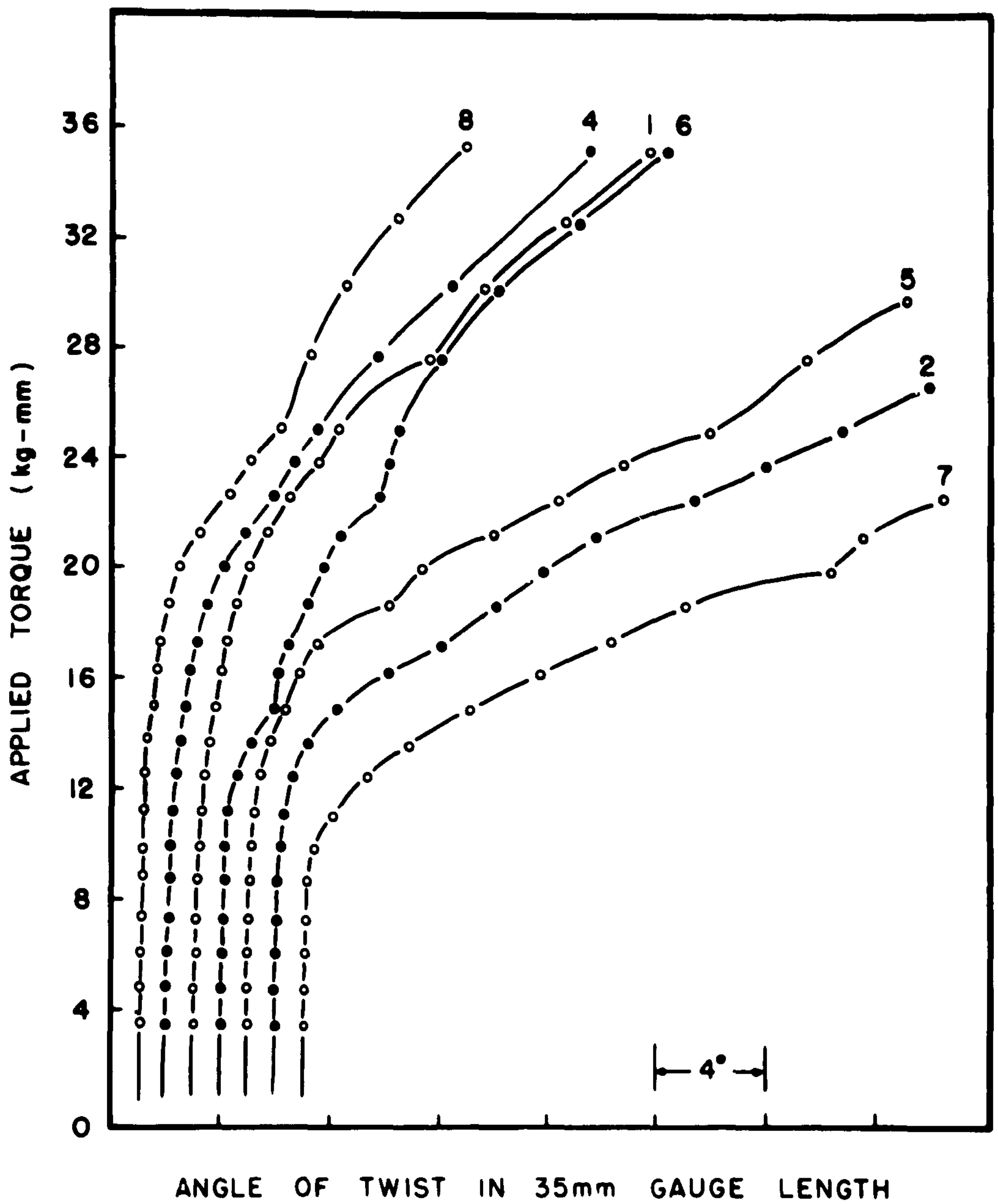

Fig. 6. Torque - twist curves for aluminum crystals.

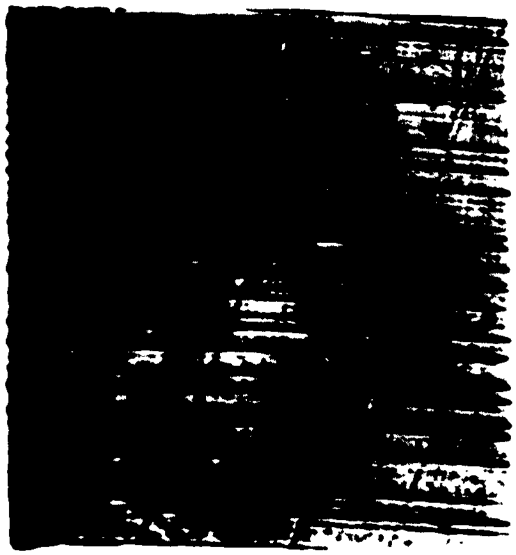

Fig. 7. Photomicrograph of the surface of crystal Al-15 at an applied torque of 30.0 kg-mm. The specimen axis is vertical. Lines due to first slip slope upwards to the left, those due to second slip are almost horizontal and those due to third slip slope upwards to the right. X 100.

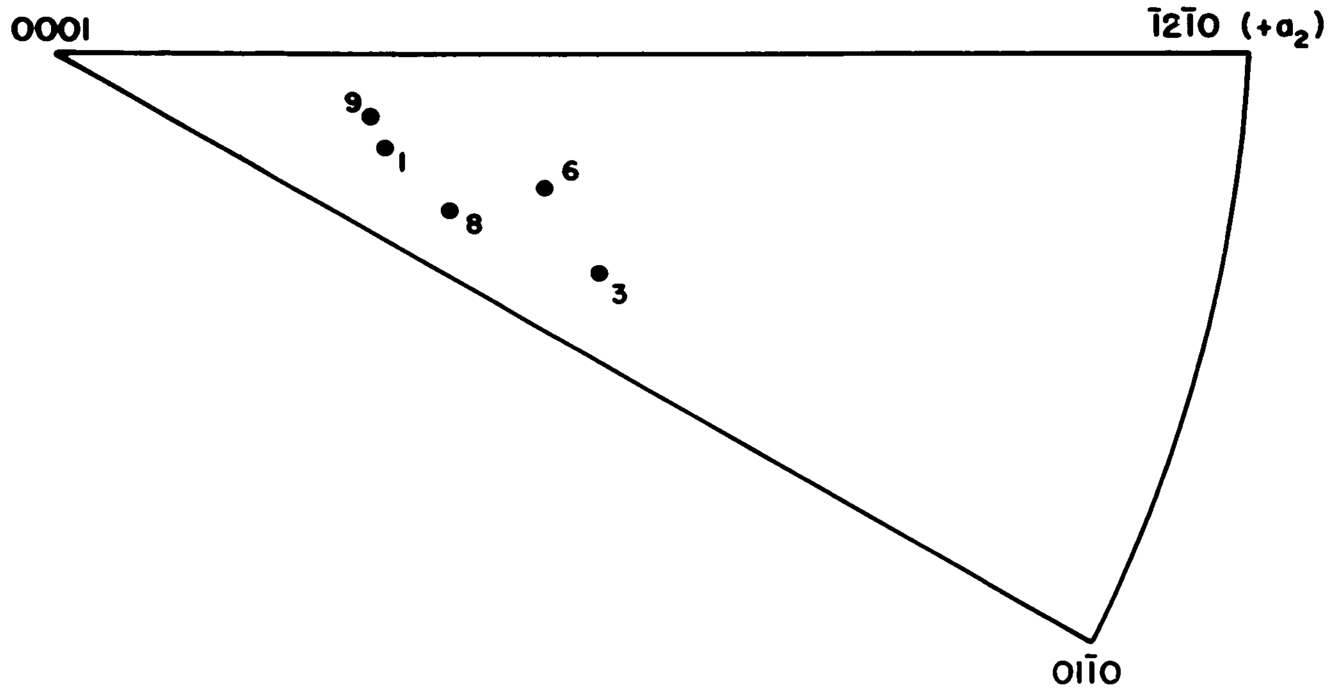

Fig. 8. Orientations of magnesium crystals.

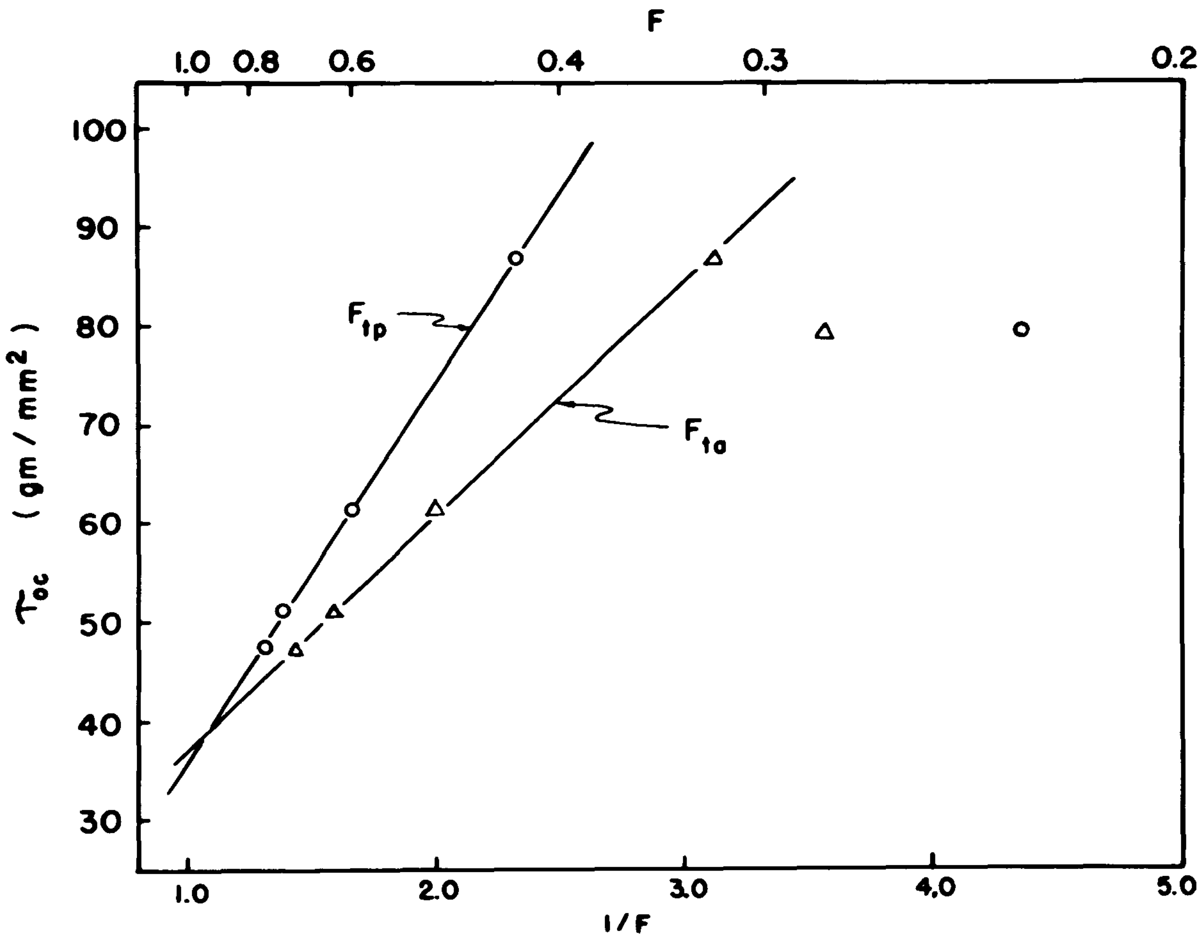

Fig. 9. Dependence of plastic yielding on orientation factors F_{tp} and F_{ta} for magnesium crystals.

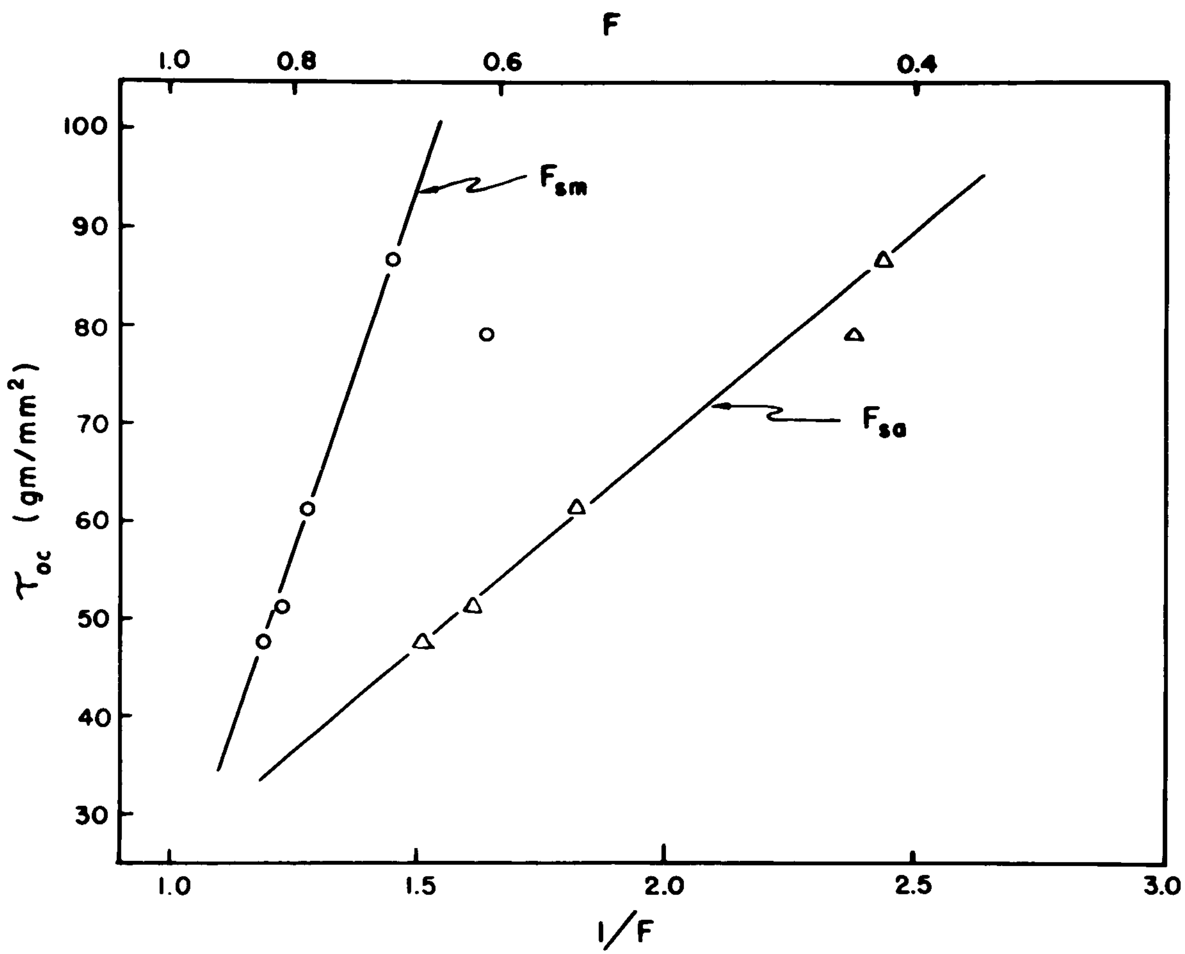

Fig. 10. Dependence of plastic yielding on orientation factors F_{sm} and F_{sa} for magnesium crystals.

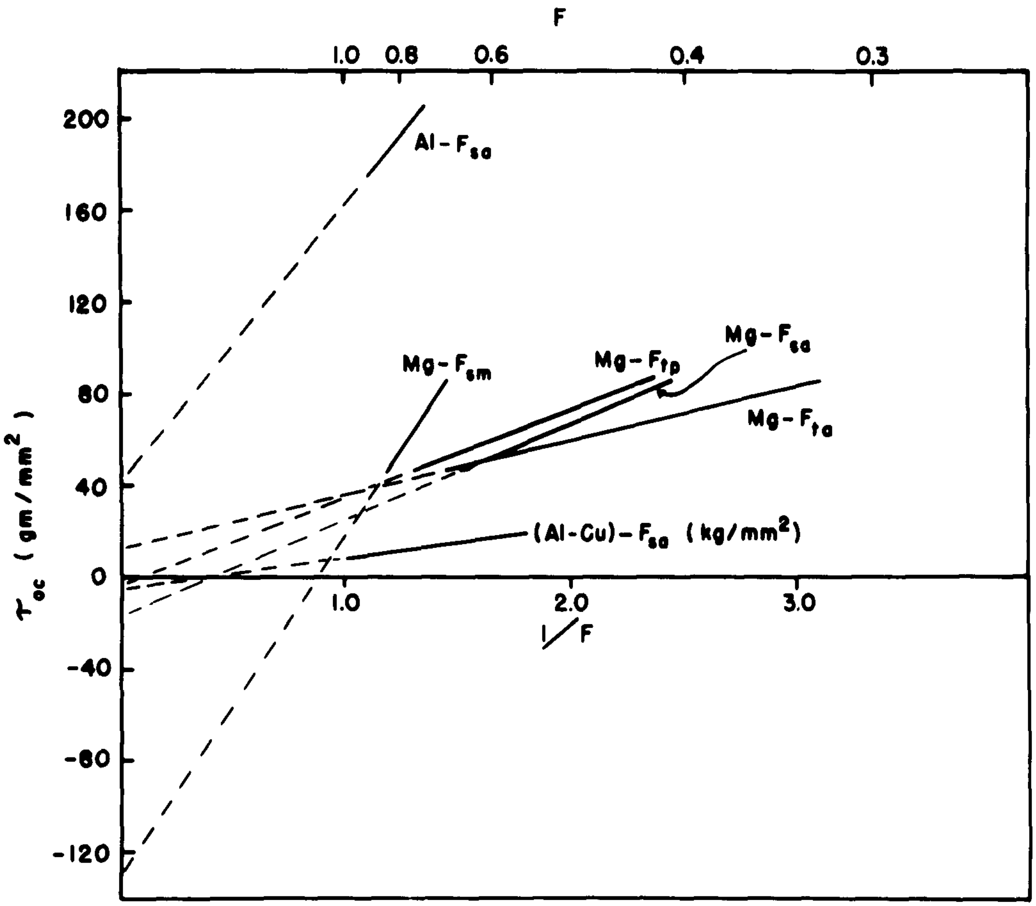

Fig. 11. Summary of the data relating torsional yielding to orientation factors. The solid lines show the ranges actually covered by the experimental results; the dashed lines are extrapolations.

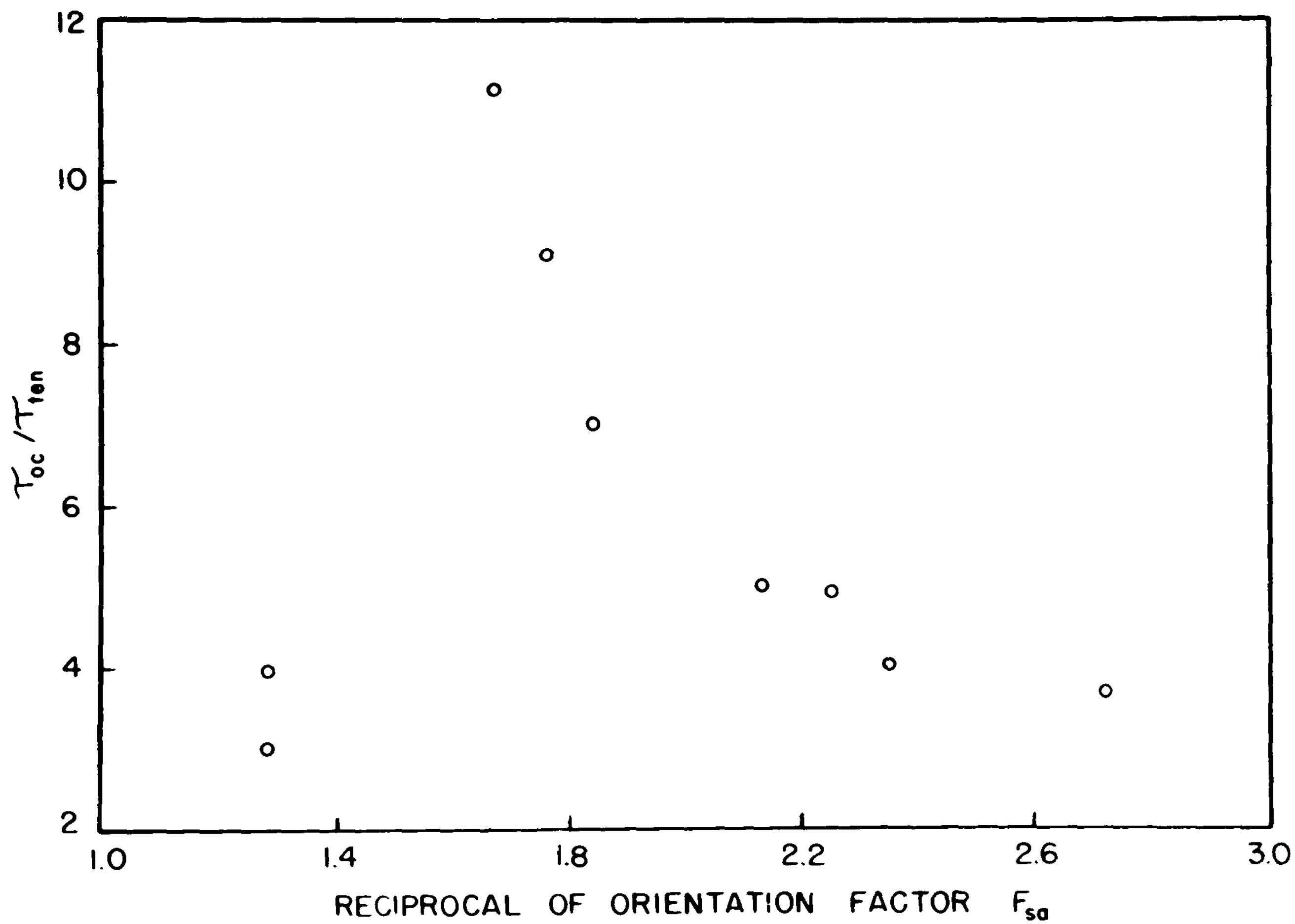

Fig. 12. **Measurements of Brown and Rosenbaum on zinc crystals.**

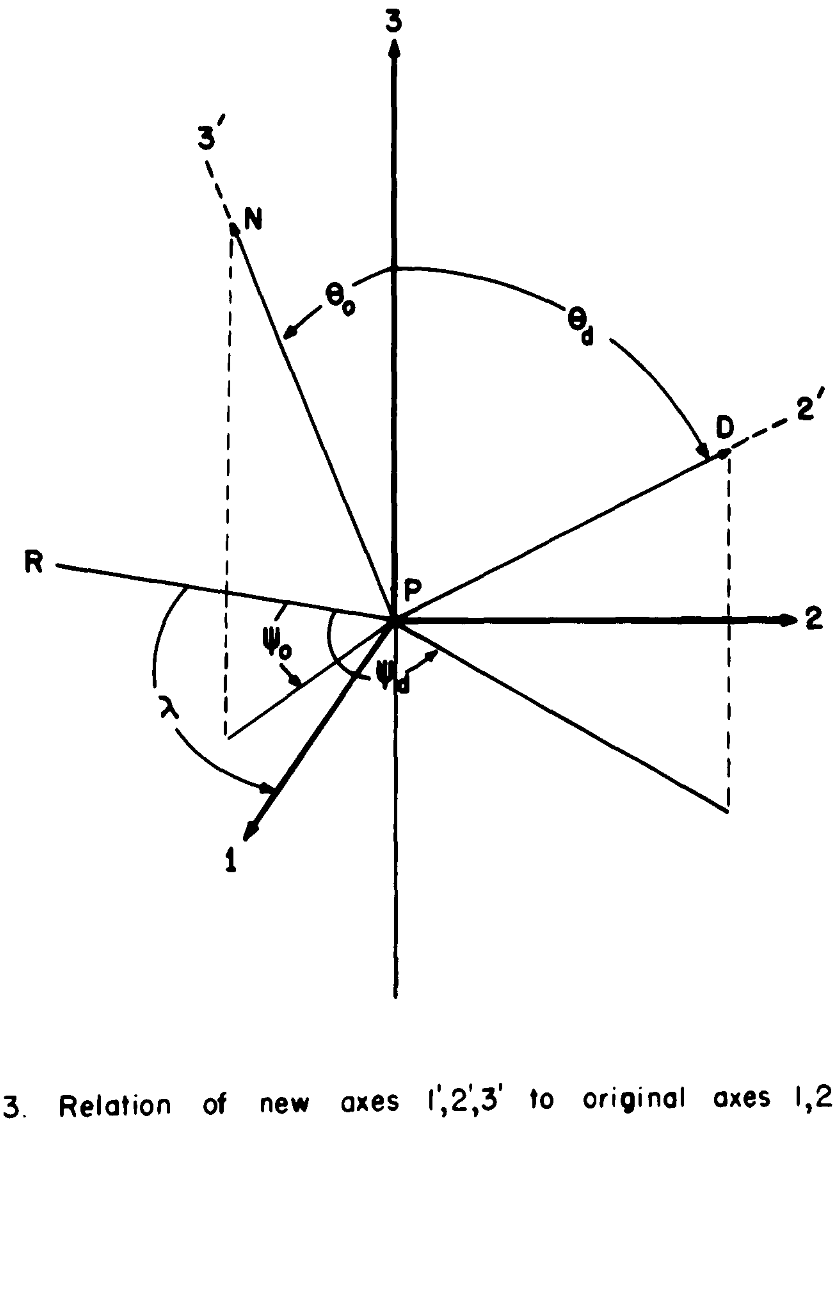

Fig. 13. Relation of new axes 1′,2′,3′ to original axes 1,2,3.

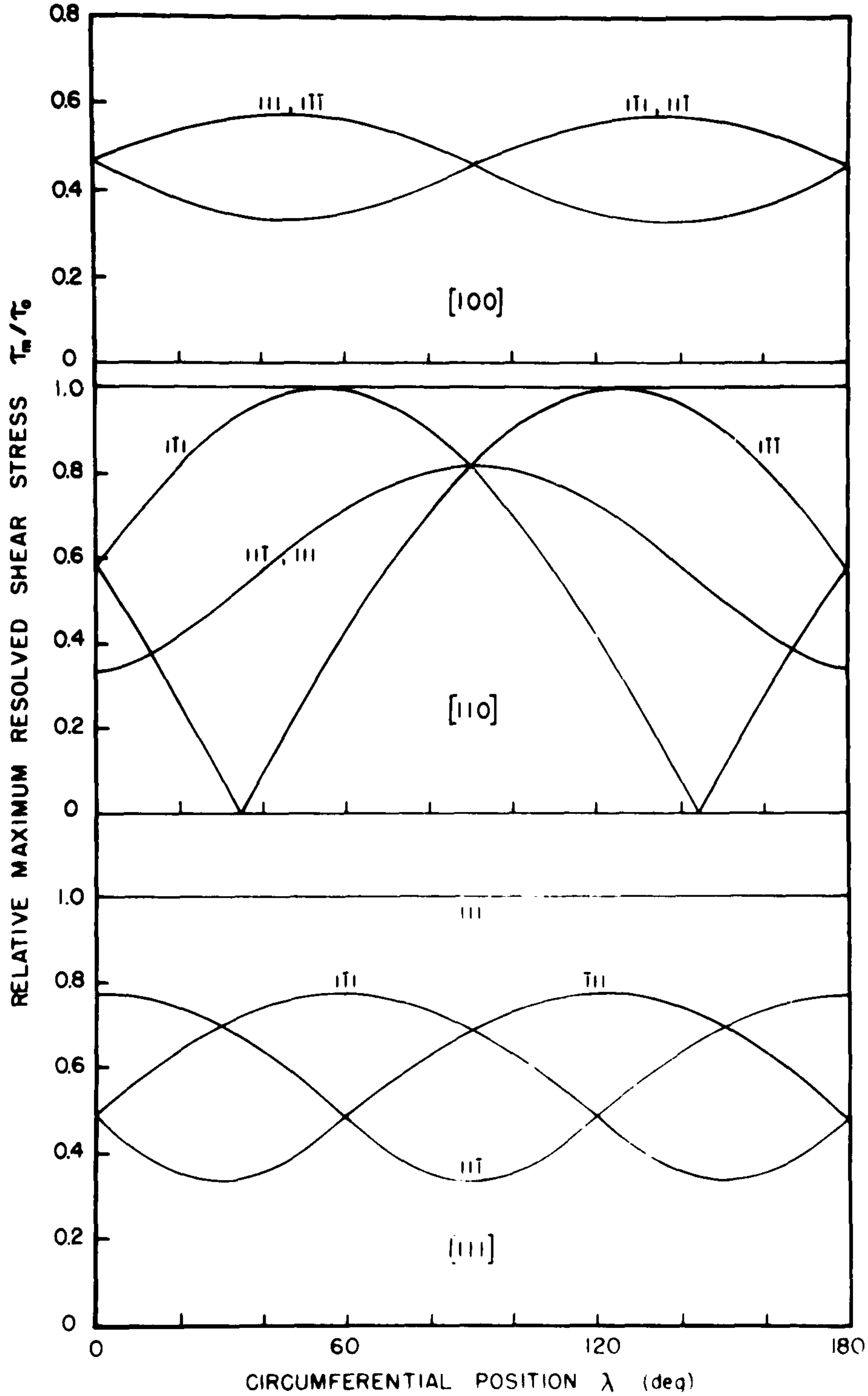

Fig. 1. Calculated variation of τ_m/τ_o with λ for [illegible] parallel to [100], [110], and [111]

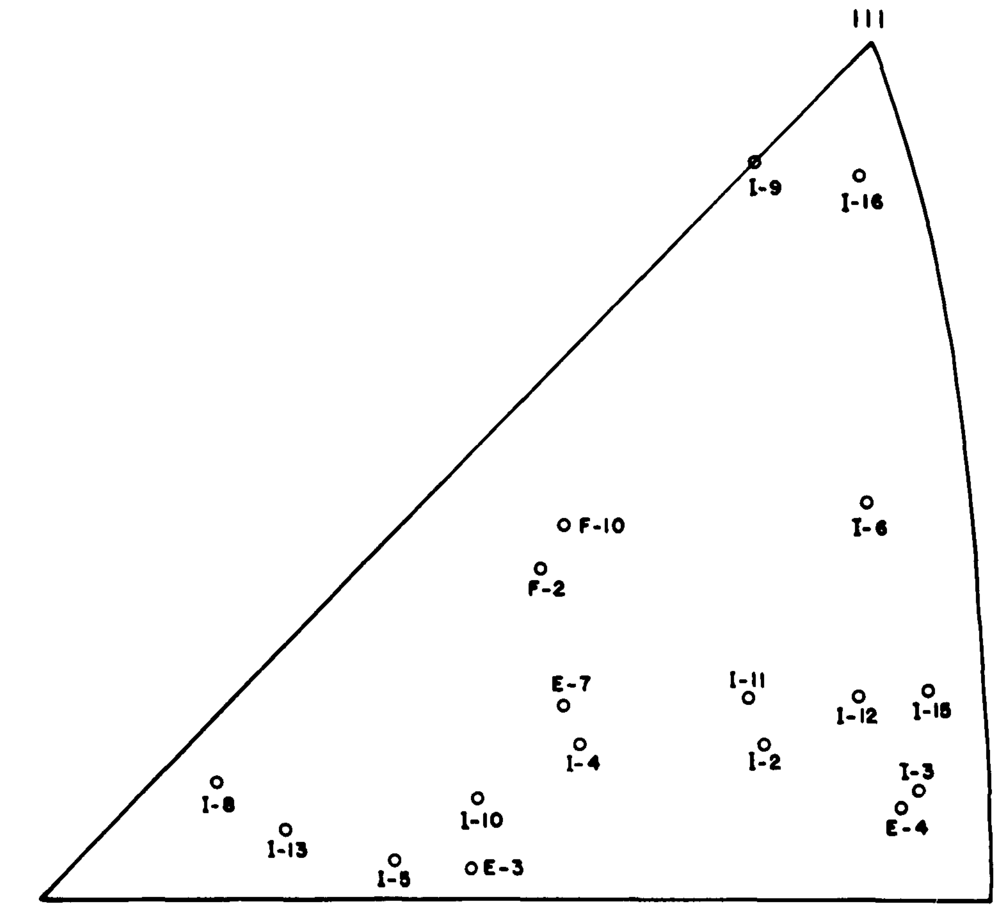

Fig. 2. Orientations of crystals tested in torsion.

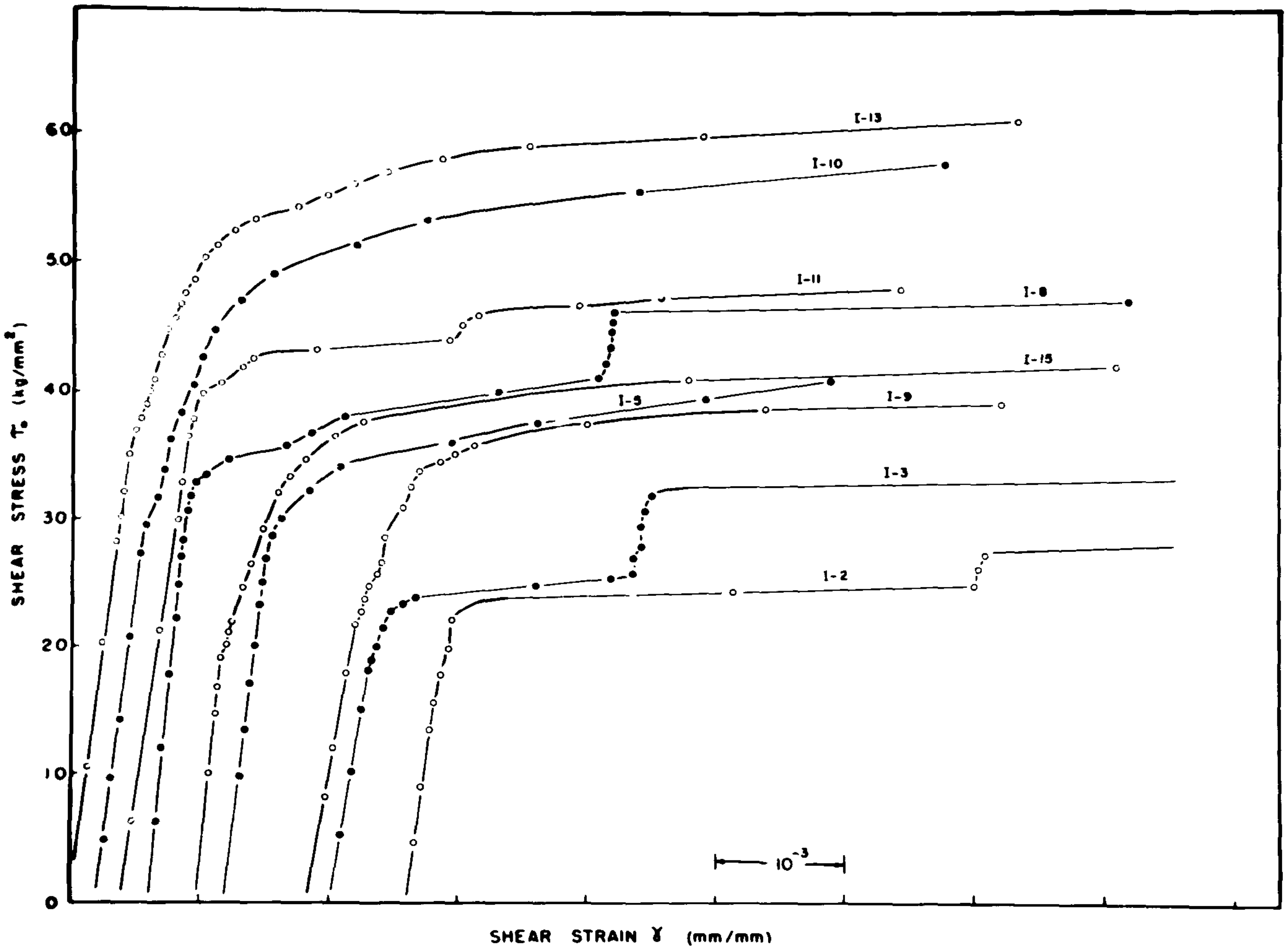

Fig.3. Typical shear stress — shear strain curves in torsion

Not corrected for machine friction.

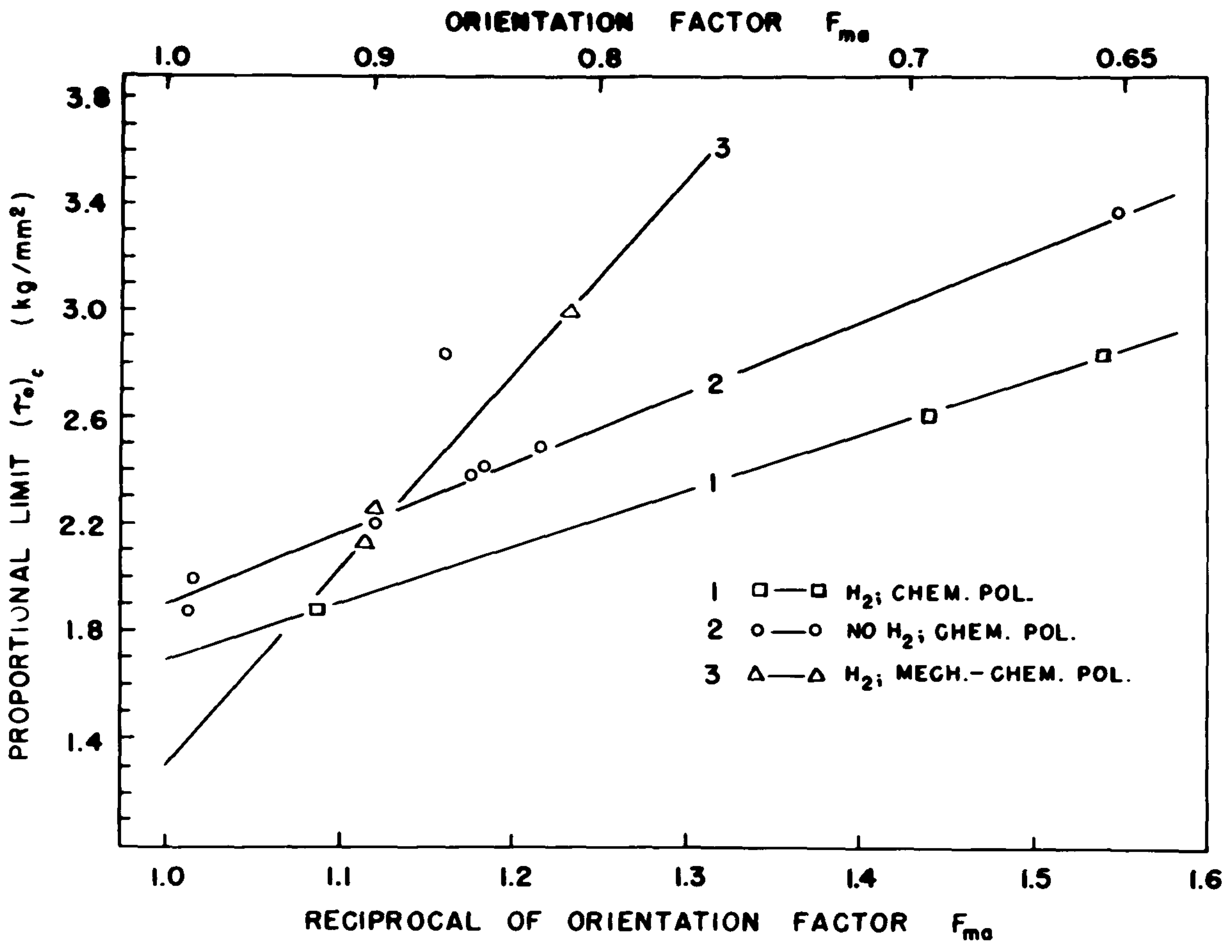

Fig. 4. Dependence of plastic yielding on orientation factor F_{ma}.

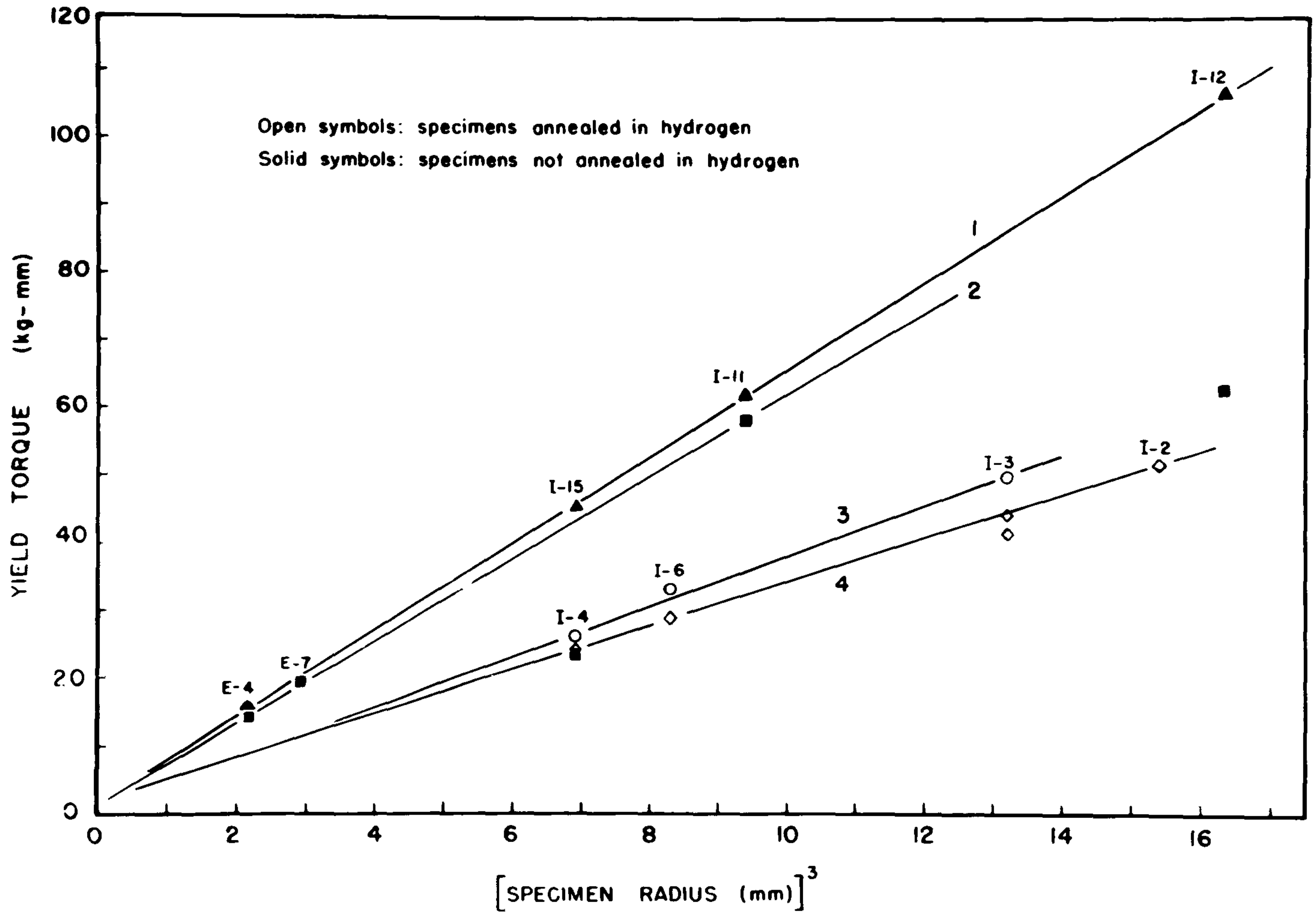

Fig 5 Variation of the torque T_c at the proportional limit (curves 2 and 4) and at "easy deformation" (curves 1 and 3) with the cube of the specimen radius.

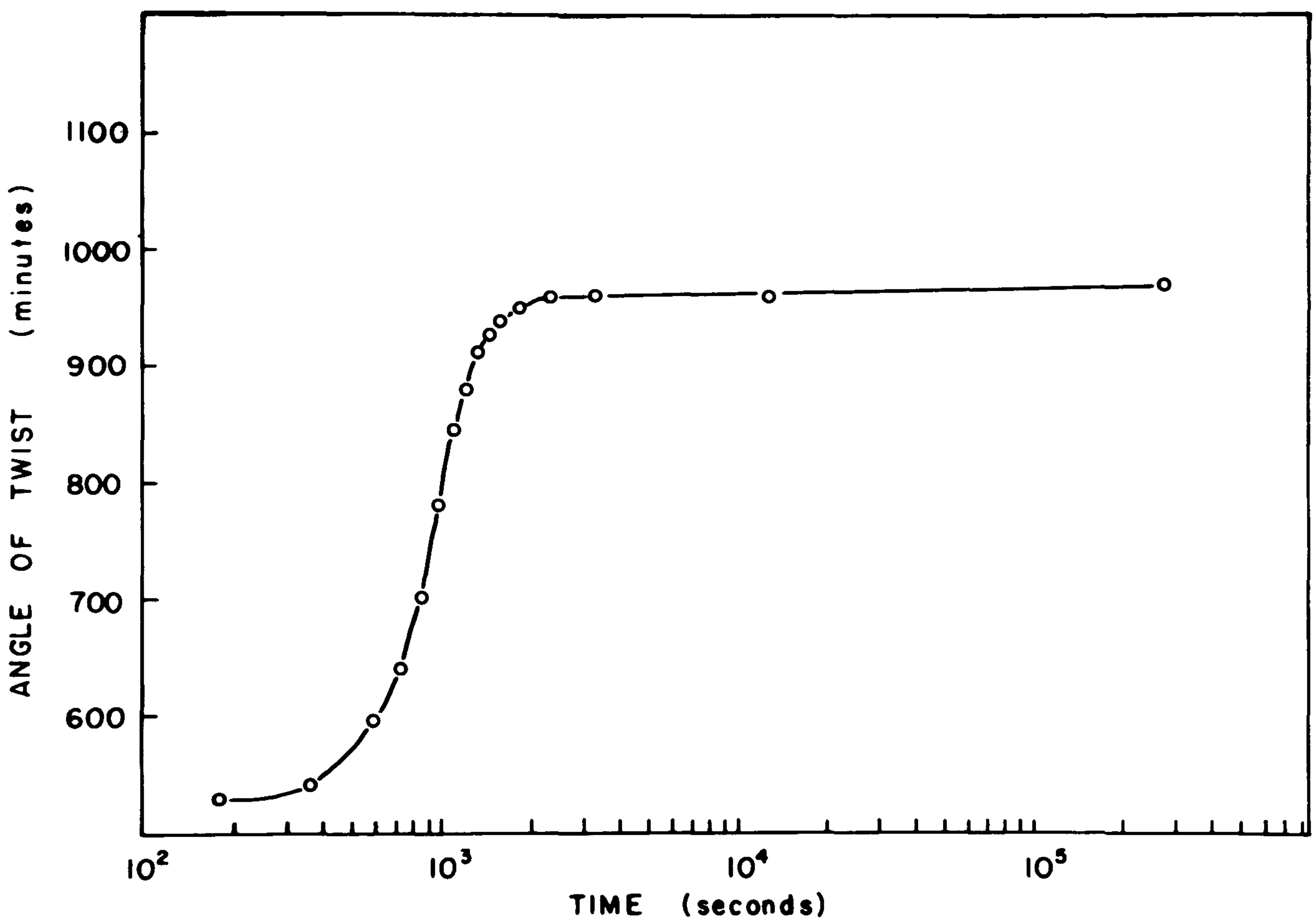

Fig. 6. Transient deformation in crystal E-3.